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# Homework 10

10/17/2021

1. Proof that there are infinitely many prime numbers:
Suppose that there is a finite number $n$ of prime numbers: $p_1 < p_2 < p_3 < \dots < p_n$.
Now take $P = p_1 p_2 p_3 \dots p_n + 1$. It is obvious that none of the prime numbers $p_1, \dots, p_n$ are factors of $P$. But every integer can be written in prime factorization, therefore either $P$ is divisible by a prime number bigger than $p_n$, or $P$ is a prime number, which is also bigger than $p_n$. In either case, this proves that there must be a prime number bigger than $p_n$. therefore there is an infinity of prime numbers. //

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2. Prime numbers less than 140

/ → 2n

\ → 3n

| → 5n

□ → 7n

O → 11n

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |
| 101 | 102 | 103 | 104 | 105 | 106 | 107 | 108 | 109 | 110 |
| 111 | 112 | 113 | 114 | 115 | 116 | 117 | 118 | 119 | 120 |
| 121 | 122 | 123 | 124 | 125 | 126 | 127 | 128 | 129 | 130 |
| 131 | 132 | 133 | 134 | 135 | 136 | 137 | 138 | 139 | 140 |

After identifying all of multiples of prime numbers up to 11 because that is the biggest integer $\leq \sqrt{140}$. and crossing them out, we have that the prime numbers are:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47 53, 59, 61, 67, 71, 73, 79, 83, 89, 91, 97, 101, 103, 107 109, 113, 127, 131, 137, 139.

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3. $3003 = 3\cdot(1001)$

$= 3\cdot(7\cdot 143) = 3\cdot 7\cdot 11\cdot 13 \rightarrow [3,1], [7,1]\ [11,1]\ [13,1]$

4. To find how many prime numbers $\leq e^{100}$ there exist use $\pi(n)$ approximation.

$$\pi(e^{100}) \approx \frac{e^{100}}{\ln e^{100}} = \frac{e^{100}}{100} \approx 2.68\cdot 10^{41}$$

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