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Homework 10

(1) Fully understand and be able to reproduce the proof that there are infinitely many primes

We will prove by contradiction. Suppose there are only finitely many primes

We can label these finitely many primes  $p_1, p_2, p_3, \dots, p_n$  for some  $n \in \mathbb{N}$ .

Next let us consider the prime  $p = p_1 p_2 p_3 \dots p_n + 1$ . This prime has a remainder

1 when divided by any prime from  $p_1, \dots, p_n$ . Hence we can say that  $p$  is

a larger prime or  $p$  is divisible by a prime larger than  $p_n$ . This contradicts our initial assumption that  $p_n$  is the largest prime.

(2) Compute from scratch all the primes  $\leq 140$ .

1 2 3 4 5 6 7 8 9 10 11 12 13 14  
15 16 17 18 19 20 21 22 23 24 25 26 27 28  
29 30 31 32 33 34 35 36 37 38 39 40 41 42  
43 44 45 46 47 48 49 50 51 52 53 54 55 56  
57 58 59 60 61 62 63 64 65 66 67 68 69 70  
71 72 73 74 75 76 77 78 79 80 81 82 83 84  
85 86 87 88 89 90 91 92 93 94 95 96 97 98  
99 100 101 102 103 104 105 106 107 108 109 110 111 112  
113 114 115 116 117 118 119 120 121 122 123 124 125 126  
127 128 129 130 131 132 133 134 135 136 137 138 139 140

2, 3, 5, 7, 11, 13, 17,  
19, 23, 29, 31, 37, 41, 43, 47,  
53, 59, 61, 67, 71, 73, 79, 83,  
89, 97, 101, 103, 107, 109, 113,  
127, 131, 137, 139

(3) Write 3003 as a product of prime powers.

$$3 \cdot (1001)$$

$$3 \cdot (11) \cdot (91)$$

$$3 \cdot (11) \cdot (7) \cdot (13)$$

$$3 \cdot (7) \cdot (11) \cdot (13)$$

(4) How many primes, roughly, are there  $\leq e^{100}$ ?

(I am not sure how to proceed).