NAME: Tianyi He
E-MAIL ADDRESS: th586@scarletmail.rutgers.edu
It is OK to post the homework in your web-site

1. Suppose there are finitely many primes, n of them, put them in order, $p_1, p_2, p_3, \cdots p_n$

Let $P = p_1 p_2 p_3 \cdots p_n + 1$

Since P leaves remainder 1 when divided by each of $p_1, \cdots, p_n$, then P is either prime (larger than p_n), or is divisible by a prime larger than p_n, a contradiction.

2. 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 137, 139

3. $3003 = 3 \times 1001 = 3 \times 11 \times 91 = 3 \times 11 \times 7 \times 13$

4. $p_n \approx n \ln n$

$n \ln n \le e^{100} \Rightarrow \quad n^n \le e^{e^{100}}$

I did not figure out how to simplify it, so I use a calculator to calculate that $n \le 2.81 \times 10^{41}$, so I thought the answer should be 2×10^{41}.