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Homework 10 10/18/2021

1. Assume to the contrary that there are finitely many primes $p_1, \dots, p_n$. Let $a = p_1 \cdots p_n + 1$. When a is divided by any of $p_1, \dots, p_n$, there is a remainder of 1 so either a is divisible by a prime not $p_1, \dots, p_n$ or it is another prime itself. In either case, it would mean there are not infinitely many primes.

2. 2, 3, ~~4~~, 5, ~~6~~, 7, ~~8~~, ~~9~~, ~~10~~, 11, ~~12~~, 13, ~~14~~, ~~15~~, ~~16~~, 17, ~~18~~, 19, ~~20~~, ~~21~~, ~~22~~, 23, ~~24~~, ~~25~~, ~~26~~, ~~27~~, ~~28~~, 29, ~~30~~, 31, ~~32~~, ~~33~~, ~~34~~, ~~35~~, ~~36~~, 37, ~~38~~, ~~39~~, ~~40~~, 41, ~~42~~, 43, ~~44~~, ~~45~~, ~~46~~, 47, ~~48~~, ~~49~~, ~~50~~, 51, ~~52~~, 53, ~~54~~, ~~55~~, ~~56~~, ~~57~~, ~~58~~, 59, ~~60~~, 61, ~~62~~, ~~63~~, ~~64~~, ~~65~~, ~~66~~, 67, ~~68~~, ~~69~~, ~~70~~, 71, ~~72~~, 73, ~~74~~, ~~75~~, ~~76~~, ~~77~~, ~~78~~, 79, ~~80~~, ~~81~~, ~~82~~, 83, ~~84~~, ~~85~~, ~~86~~, ~~87~~, ~~88~~, 89, ~~90~~, ~~91~~, ~~92~~, ~~93~~, ~~94~~, ~~95~~, ~~96~~, 97, ~~98~~, ~~99~~, ~~100~~, 101, ~~102~~, 103, ~~104~~, ~~105~~, ~~106~~, 107, ~~108~~, 109, ~~110~~, ~~111~~, ~~112~~, 113, ~~114~~, ~~115~~, ~~116~~, ~~117~~, ~~118~~, 119, ~~120~~, ~~121~~, ~~122~~, ~~123~~, ~~124~~, ~~125~~, ~~126~~, 127, ~~128~~, ~~129~~, ~~130~~, 131, ~~132~~, 133, ~~134~~, ~~135~~, ~~136~~, 137, ~~138~~, 139, ~~140~~

2, 3, 5, 7, 11, 13, 17, 19, 29, 31, 37, 41, 43, 51, 53, 59, 61, ~~67~~, 71, 79, 89, 97, 101, 103, 107, 109, 113, 119, 127, 131, 137, 139

3. $3003 = [3, 1]$ $\quad \frac{3003}{3} = 1001$

$1001 = [7, 1]$ $\quad 1001/7 = 143$

$143 = [11, 1]$ $\quad \frac{143}{11} = 13$

so $3003 = 3 \times 7 \times 11 \times 13$

4. $$\frac{e^{100}}{\log(e^{100})}$$