Quin Buob

HW 10 OK to Post

1) Assume there are finitely many primes:

$[p_1, p_2, \dots, p_k]$

There exists a number P which:

$P = p_1 p_2 p_3 \cdots p_k + 1$

This leaves a remainder 1 when divided and prime number.

By the fundamental Thm of Arithmetic:

P must either be prime or be divisible by a prime number larger than p_k - which is a contradiction. So there must be infinitely many primes.

2) to find the primes ≤ 140

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139

3) 3,003

$L(3003) = [3, 1], L\left(\frac{3003}{3}\right)$

$L\left(\frac{3003}{3} = 1001\right) = [7, 1], L\left(\frac{1001}{7}\right)$

$L\left(\frac{1001}{3} = 143\right) = [11, 1], L\left(\frac{143}{11}\right)$

$L\left(\frac{143}{11} = 13\right) = [13, 1]$

$L(3003) = [3, 1], [11, 1], [13, 1], [7, 1]$

$3003 = 3 \times 7 \times 11 \times 13$

How many primes are there $\leq e^{100}$

$$Pi(e^{100}) = \frac{e^{100}}{\ln^{e^{100}}} = \frac{e^{100}}{100}$$

There are $e^{100}/100$ primes $\leq e^{100}$