Larry Vo
okay

H.W 10

1. Suppose there are a finite number of primes. Then each number can be written as a product of primes so let $P = P_1 \cdot P_2 \cdots P_n$ where $P_k \in \{P_1, P_2, \dots, P_n\}$ are distinct primes. Now lets look at the next number $P_1 \cdot P_2 \cdots P_n + 1$, if it is prime then we are done and there is a infinite number of primes, if it is composite then it can be divided by some $P_k \in \{P_1, P_2, \dots, P_n\}$. Since P_k is a factor of $P_1 \cdot P_2 \cdots P_n + 1$ and $P = P_1 \cdot P_2 \cdots P_n$ then it is a factor of $P_1 \cdot P_2 \cdots P_n + 1 - P = 1$, Since P_k is a factor then $\frac{P_1 \cdot P_2 \cdots P_n + 1 - P}{P_k} = \frac{1}{P_k}$

but the left hand side is an integer since we are factoring out a P_k while the RHS is not an integer thus the contradiction. Therefore there are an infinite amount of primes.

2\. 2 3 ~~4~~ 5 ~~6~~ 7 ~~8~~ ~~9~~ ~~10~~ 11 ~~12~~ 13
~~14~~ ~~15~~ ~~16~~ 17 ~~18~~ 19 ~~20~~ 21 ~~22~~ 23 ~~24~~ ~~25~~
~~26~~ ~~27~~ ~~28~~ 29 ~~30~~ 31 ~~32~~ 33 ~~34~~ ~~35~~ ~~36~~ 37
~~38~~ ~~39~~ ~~40~~ 41 ~~42~~ 43 ~~44~~ ~~45~~ ~~46~~ 47 ~~48~~ ~~49~~
~~50~~ ~~51~~ ~~52~~ 53 ~~54~~ ~~55~~ 56 57 ~~58~~ 59 ~~60~~ 61
~~62~~ ~~63~~ ~~64~~ ~~65~~ ~~66~~ 67 ~~68~~ ~~69~~ ~~70~~ 71
~~72~~ 73 ~~74~~ ~~75~~ ~~76~~ ~~77~~ ~~78~~ 79 ~~80~~
~~81~~ ~~82~~ 83 ~~84~~ ~~85~~ ~~86~~ ~~87~~ ~~88~~ 89 ~~90~~ ~~91~~ ~~92~~
~~93~~ ~~94~~ ~~95~~ ~~96~~ 97 ~~98~~ ~~99~~ ~~100~~ 101 ~~102~~ 103
~~104~~ ~~105~~ ~~106~~ 107 ~~108~~ 109 ~~110~~ ~~111~~ ~~112~~ 113 ~~114~~
~~115~~ ~~116~~ ~~117~~ ~~118~~ ~~119~~ ~~120~~ ~~121~~ ~~122~~ ~~123~~
~~124~~ ~~125~~ ~~126~~ 127 ~~128~~ ~~129~~ ~~130~~
131 ~~132~~ ~~133~~ ~~134~~ ~~135~~ ~~136~~ 137 ~~138~~
139 ~~140~~.

1. cross out all factors of 2 besides 2
2. cross out all factor of 3 besides 3
3. cross out all factors of 5 besides 5
4. cross out all factors of 7 besides 7
5. cross out all factors of 11 besides 11
6. Cross out all factors of 13 besides 13

The smallest survior 13 is larger than $\sqrt{140}$ so all the numbers not crossed out are prime numbers.

3. $3003 = 3 \cdot 1001 = 3 \cdot 11 \cdot 91 = 3 \cdot 11 \cdot 13 \cdot 7$

4. $\pi(e^{100}) = \dfrac{e^{100}}{\log(e^{100})}$

there is roughly $\dfrac{e^{100}}{\log(e^{100})}$ number of primes $\leq e^{100}$

Scanned with CamScanner