Homework 1

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1. proof: If $f(x) = (0,0)$, which means the reminder is 0, when x is divided by n_1 and also by n_2

Because n_1 and n_2 are relatively prime,

x should be a multiple of $n_1 n_2$ or $x = 0$.

Since $0 \le x < n_1 n_2$, $x = 0$.

Then if $f(x_1) = f(x_2)$, $f(x_1 - x_2) = 0$, so $x_1 - x_2 = 0$, $x_1 = x_2$

So f is one-to-one.

And since $[0, n_1 n_2 - 1]$ and $[0, n_1 - 1] \times [0, n_2 - 1]$ have the same number of elements and f is one-to-one, then it must be onto.

Since f is one-to-one and onto, it is bijection

2. $\frac{1}{2} + \frac{1}{3} + \frac{1}{5} = \frac{31}{30}$

$\frac{31}{30} = \frac{1}{2} + \frac{1}{5} + \frac{1}{6} + \frac{1}{10} + \frac{1}{15}$

In dividing pizzas, $\frac{1}{2} + \frac{1}{3} + \frac{1}{5}$ is better because it is easier to divide less times.

3. $f(x) := (x \pmod 3, x \pmod 7)$

$f(1) = (1,1)$, $f(2) = (3,3)$, $f(3) = (0,3)$, $f(4) = (1,4)$, $f(5) = (2,5)$

$f(6) = (0,6)$, $f(7) = (1,0)$, $f(8) = (2,1)$, $f(9) = (0,2)$, $f(10) = (1,3)$

$f(11) = (2,4)$ $f(12) = (0,5)$ $f(13) = (1,6)$ $f(14) = (2,0)$ $f(15) = (0,1)$

$f(16) = (1,2)$ $f(17) = (2,3)$ $f(18) = (0,4)$ $f(19) = (1,5)$ $f(20) = (2,6)$

(a) 20

(b) 4

(c) 9