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Homework for Lecture 1

① Suppose $f(x) = (0, 0)$. This means that the remainders after dividing by n_1 and n_2 are both 0. Thus x is divisible by n_1 and n_2. We know that n_1 and n_2 are relatively prime, so this means that x is a multiple of $n_1 n_2$. Also, $0 \leq x < n_1 n_2$, so x can not be a multiple of $n_1 n_2$ that is larger than $n_1 n_2$. Therefore $x = 0$.

Next we prove that the mapping is one-to-one. We will show that if $f(x_1) = f(x_2)$, then $x_1 = x_2$. If $f(x_1) = f(x_2)$, this means that $f(x_1 - x_2) = 0$, so $x_1 - x_2 = 0$. Therefore $x_1 = x_2$, showing the mapping is one-to-one.

Finally, we show that the mapping is onto. We see that $[0, n_1 n_2 - 1]$ contains the same number of elements as $[0, n_1 - 1] \times [0, n_2 - 1]$. Since they have an equivalent number of elements and we know that the mapping is one-to-one, then the mapping is onto as well.

Thus, the mapping is one-to-one and onto (a bijection).

② $\frac{1}{2}+\frac{1}{3}+\frac{1}{5}=\frac{31}{30}=1\frac{1}{30}$

Let $x=1$. Then $\frac{1}{x}=1$. $\text{Ceil}\left(\frac{1}{x}\right)=2$.

$$EF(1)=\frac{1}{2}+EF\left(1-\frac{1}{2}\right)$$

$=\frac{1}{2}+EF\left(\frac{1}{2}\right)$ We continue the process since $\frac{1}{2}$ is a duplicate

$\text{Ceil}(2)=3$

$$=\frac{1}{2}+\frac{1}{3}+EF\left(\frac{1}{2}-\frac{1}{3}\right)$$

$$=\frac{1}{2}+\frac{1}{3}+EF\left(\frac{1}{6}\right)$$

$$=\frac{1}{2}+\frac{1}{3}+\frac{1}{6}$$

So $\frac{31}{30}=1\frac{1}{30}=\frac{1}{2}+\frac{1}{3}+\frac{1}{6}+\frac{1}{30}$

Method 1: $\frac{31}{30}=\frac{1}{2}+\frac{1}{3}+\frac{1}{5}=\frac{15+10+6}{30}$

Divide the first 15 pizzas into two equal halves and give them to the 30 people. Then, divide the next 10 pizzas into three equal parts and give them to the 30 people. Take the last 6 pizzas and divide them into five equal parts and give them to the 30 people.

Method 2: $\frac{31}{30}=\frac{1}{2}+\frac{1}{3}+\frac{1}{6}+\frac{1}{30}=\frac{15+10+5+1}{30}$

Divide the first 15 pizzas into two equal halves and give them to the 30 people. Then, divide the next 10 pizzas into three equal parts and give them to the 30 people. Divide the next 5 pizzas into six equal parts and give them to the 30 people. Divide the last pizza into thirty equal parts and give them to the 30 people.

Method 1 is better because in Method 2, it is harder to divide a pizza into thirty equal parts.

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③ a.) Define $f(x) = (x \pmod 3, x \pmod 7)$

$f(0) = (0,0)$	$f(5) = (2,5)$	$f(10) = (1,3)$	$f(15) = (0,1)$	$\boxed{f(20) = (2,6)}$
$f(1) = (1,1)$	$f(6) = (0,6)$	$f(11) = (2,4)$	$f(16) = (1,2)$	
$f(2) = (2,2)$	$f(7) = (1,0)$	$f(12) = (0,5)$	$f(17) = (2,3)$	
$f(3) = (0,3)$	$f(8) = (2,1)$	$f(13) = (1,6)$	$f(18) = (0,4)$	
$\boxed{f(4) = (1,4)}$	$\boxed{f(9) = (0,2)}$	$f(14) = (2,0)$	$f(19) = (1,5)$	

20 is the smallest integer, so the second smallest integer is $20 + (3 \cdot 7) = 41$

b.) Using the table above, we see that 4 is the smallest integer, since $f(4) = (1,4)$. The second smallest integer is $4 + (3 \cdot 7) = 25$.

c.) Using the table above, we see that 9 is the smallest integer, since $f(9) = (0,2)$. The second smallest integer is $9 + (3 \cdot 7) = 30$.