Sarah Catovic HW 1 9/12/2021

Chapter II Sections 1-3 Summary: Societies lived around rivers since areas with rivers pointed to fertile land which could be used to grow crops. Some of the more well known societies such as the Mesopotamians and Egyptians got their origin this way, but did not seem, for the most part, to develop beyond basing their economy on agriculture. However they did employ mathematics to understand area as well as date and time. One of the ideas that came up was the use of fractions by the Egyptians, probably to understand dividing things such as bread amongst people. They used fractions which had one in its numerator, probably due to its simplicity. We also see the emergence of figuring out the area of different shapes, which proved a useful tool for farming. Though some of these ancient oriental societies did employ different mathematical techniques, they were still an underdeveloped society.

1. Firstly, it is assumed that $f(x) = (0,0)$, which means that in the when dividing x by n_1 there is no remainder and dividing x by n_2 there is no remainder. Since n_1 and n_2 are relatively prime, then x must be some multiple of $n_1 n_2$. But, it has been already established that $0 \le x < n_1 n_2$ so the only way to get a remainder of 0 when x is divided by both n_1 and n_2 is for x to be 0.

2. The Egyptian fraction $\frac{1}{2} + \frac{1}{3} + \frac{1}{5}$ as a regular fraction is $\frac{15}{30} + \frac{10}{30} + \frac{6}{30} = \frac{31}{30}$. Using the greedy algorithm to convert it to another Egyptian Fraction, firstly let $x = \frac{31}{30}$ so $\frac{1}{x} = \frac{30}{31}$, the ceiling of $\frac{1}{x}$ is 1 so Egyptian Fraction$\left(\frac{31}{30}\right) = \frac{1}{1}$ − Egyptian Fraction$\left(\frac{31}{30} - \frac{1}{1}\right)$ $= \frac{1}{30}$ so the Egyptian Fraction is thus $\frac{1}{1} + \frac{1}{30}$.

Thinking about it this way, dividing 31 pizzas among 30 people, it's quite difficult to cut a pizza into 30 pieces. For $\frac{1}{2} + \frac{1}{3} + \frac{1}{5}$, it's easier to divide pizzas into halves, thirds, and fifths. Though with $\frac{1}{1} + \frac{1}{30}$ each gets a whole pie, the difficulty is in $\frac{1}{30}$ of the pizza.

3. (a) $x \equiv 2 \pmod 3$; $x \equiv 6 \pmod 7$

$f(x) = (x \pmod 3, x \pmod 7)$

$f(1) = (1,1)$, $f(2) = (2,2)$, $f(3) = (0,3)$, $\underline{f(4) = (1,4)}$,

$f(5) = (2,5)$, $f(6) = (0,6)$, $f(7) = (1,0)$, $f(8) = (2,1)$; $f(9) = (0,2)$,

$f(10) = (1,3)$, $f(11) = (2,4)$, $f(12) = (0,5)$, $f(13) = (1,6)$, $f(14) = (2,0)$

$f(15) = (0,1)$, $f(16) = (1,2)$, $f(17) = (2,3)$, $f(18) = (0,4)$,

$f(19) = (1,5)$, $\underline{f(20) = (2,6)}$

The smallest integer is 20 and the second smallest is $20 + 3 \times 7 = 41$.

(b) The smallest integer according to the above list is 4 and the second smallest is $4 + 3 \cdot 7 = 25$.

(c) The smallest integer according to the above list is 9 and the second smallest integer is $9 + 3 \cdot 7 = 30$