HOMEWORK #1 - HISTORY OF MATH

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① In order to use the Chinese remainder theorem, we have to make sure that the divisors are relatively prime with each other. This means that the GCD's must be equal to 1.

- Next, we want to define $f(x) := (x \pmod{n_1}, x \pmod{n_2}, \dots x \pmod{n_k})$ and construct a chart ~~of~~ plugging in various x values.
- ~~The bijection mapps This maps to~~ We create a table by selecting ~~a~~ a closed interval between two numbers to select the range in which we want our x value to exist. ~~Clear this range~~ We get this range by multiplying our ~~Then, we create a mapping where~~ divisors and subtracting by 1. So our closed interval is $[0, (n_1 n_2 \cdot \dots \cdot n_k) - 1]$
- Our map will look like $[0, (n_1 \cdot n_2 \cdot \dots \cdot n_k) - 1] \to [0, n_1 - 1] \cdot [0, n_2 - 1] \dots [0, n_k - 1]$. This is a bijection.
- After creating our chart listing all the $f(x) := (x \pmod{n_1}, \dots, x \pmod{n_k})$ between the X values of $[0, (n_1 \cdot n_2 \cdot \dots \cdot n_k) - 1]$ we can solve the problem.

②

$$\frac{1}{2} + \frac{1}{3} + \frac{1}{5} = \frac{31}{30} \Rightarrow EF\left(\frac{31}{30}\right) = x \Rightarrow \frac{1}{x} = \frac{30}{31} \quad \cancel{\text{ceil}\left(\frac{30}{31}\right) = 0}$$

$$x = \frac{31}{30} = 1\frac{1}{30} \Rightarrow \boxed{\frac{1}{2} + \frac{1}{2} + \frac{1}{30}}$$

- In the context of dividing pizzas, the $\frac{1}{2} + \frac{1}{3} + \frac{1}{5}$ method is the best way to divide the pizza

③ (a) $f(x) := (x \pmod 3, x \pmod 7) \Rightarrow [0, 20]$

| | | | |
|---|---|---|---|
| $f(0) = (0,0)$ | $f(5) = (2,5)$ | $f(10) = (1,3)$ | $f(15) = (0,1)$ |
| $f(1) = (1,1)$ | $f(6) = (0,6)$ | $f(11) = (2,4)$ | $f(16) = (1,2)$ |
| $f(2) = (2,2)$ | $f(7) = (1,0)$ | $f(12) = (0,5)$ | $f(17) = (2,3)$ |
| $f(3) = (0,3)$ | $f(8) = (2,1)$ | $f(13) = (1,6)$ | $f(18) = (0,4)$ |
| $f(4) = (1,4)$ | $f(9) = (0,2)$ | $f(14) = (2,0)$ | $f(19) = (1,5)$ |
| | | | $f(20) = (2,6)$ |

$$\boxed{x_1 = 20 \quad x_2 = 41}$$

| | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | 42 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 3 | 0 | 1 | 2 | 0 | 1 | 2 | 0 | 1 | 2 | 0 | 1 | 2 | 0 | 1 | 2 | 0 | 1 | 2 | 0 | 1 | 2 | 0 |
| 7 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 0 |

(b) $f(x) := (x \pmod 3,\ x \pmod 7)$; $f : (\cancel{1}, 4) \Rightarrow [0, 20]$

$$\boxed{x_1 = 4 \qquad x_2 = 25}$$

(c) $\boxed{x_1 = 9 \qquad x_2 = 51}$

| | 43 | 44 | | | | 48 | . | . | 51 | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 3 | 1 | 2 | 0 | 1 | 2 | $\boxed{0}$ | 1 | 2 | $\boxed{0}$ | 1 | 2 | 0 | 1 | 2 |
| 7 | 1 | 2 | 3 | 4 | 5 | $\boxed{6}$ | 0 | 1 | $\boxed{2}$ | 3 | 4 | 5 | 6 | 0 |