0 Chapter II summary:

The second chapter of this book continues to discuss newer advancements in society. Specifically it discusses what communities that lived along the Nile, Tigris and Euphates, Indus, Ganges, Hoang-ho, and Yang-tse did to improve their lives. One thing these communities had to do was control flooding from the river and drain swamps in order to grow abundant crops. They did this by regulating water supply with a central system of administration. This administration task was given to a specific group of people, which led to the creation of social classes. With time, Oriental mathematics originated as a science to allow computation of the calendar, administration of the harvest, organization of public works, and collection of taxes. This started with arithmetics, which evolved into algebra, which evolved into geometry.

Egyptian mathematical knowledge is derived from two mathematical papyri, the Papyrus Rhind, and the Moscow Papyrus. The problems in thes ancient papyri are quite similar to ones from more recent times. The Egyptians developed an arithmetic which focused on addition, including reducing multiplication to repeated additions. Many problems were simple and stayed within a linear equation with a single unknown. The most impressive aspect of Egyptian arithmetic was the calculus of fractions. All fractions were reduced to unit fractions. Some other problems were of more a geometric nature which led to formulas calculating area and volume

Homework 1

(1) Proof of the Chinese Remainder Theorem:

$f(x) = (x \bmod n_1, \; x \bmod n_2)$

If $f(x) = (0, 0)$, x is divisible by n_1 and n_2

Since the theorem states n_1 and n_2 are relatively prime,

- it is a multiple of n_1, n_2
- since $0 \leq x \leq n_1, n_2$, $x = 0$

Thus, $f(x_1 - x_2) = 0$ if $f(x_1) = f(x_2)$. This also means $x_1 - x_2 = 0$, so $x_1 = x_2$. This means f is one-to-one

Additionally, since $[0, n_1 n_2 - 1]$ has the same size as $[0, n_1 - 1] \times [0, n_2 - 1]$, and it is one-to-one, it must be onto, which means it has an inverse

(2) $\frac{1}{2} + \frac{1}{3} + \frac{1}{5} = \frac{15}{30} + \frac{10}{3} + \frac{6}{30} = \frac{31}{30}$

Greedy Algorithm of $\frac{31}{30}$:

$$\frac{1}{\left(\frac{30}{31}\right)} = \frac{1}{1.03} = \underline{\frac{1}{2}} \Rightarrow \frac{31}{30} - \frac{1}{2} = \frac{16}{30} \text{ or } \frac{8}{15}$$

$$\frac{1}{\left(\frac{15}{8}\right)} = \frac{1}{0.53} = \underline{\frac{1}{2}} \Rightarrow \frac{8}{15} - \frac{1}{2} = \underline{\frac{1}{30}}$$

$\frac{31}{30} = \frac{1}{2} + \frac{1}{2} + \frac{1}{30}$ versus $\frac{31}{30} = \frac{1}{2} + \frac{1}{3} + \frac{1}{5}$

The original representation of $\frac{31}{30}$ is better because dividing a pizza into five equal parts is better than dividing into thirty

③ (a) $x_1, x_2 \bmod 3 = 2$
$x_1, x_2 \bmod 7 = 6$

$f(1) = (1, 1)$	$f(2) = (2, 2)$	$f(3) = (0, 3)$
$f(4) = (1, 4)$	$f(5) = (2, 5)$	$f(6) = (0, 6)$
$f(7) = (1, 0)$	$f(8) = (2, 1)$	$f(9) = (0, 2)$
$f(10) = (1, 3)$	$f(11) = (2, 4)$	$f(12) = (0, 5)$
$f(13) = (1, 6)$	$f(14) = (2, 0)$	$f(15) = (0, 1)$
$f(16) = (1, 2)$	$f(17) = (2, 3)$	$f(18) = (0, 4)$
$f(19) = (1, 5)$	$f(20) = (2, 6)$	

$20 + (3 \cdot 7) = 41 \rightarrow \boxed{20, 41}$

(b) $x_1, x_2 \bmod 3 = 1$
$x_1, x_2 \bmod 7 = 4$

From the table created in (a) we see that the smallest x for which $f(x) = (1, 4)$ is $x = 4$
$4 + (3 \cdot 7) = 25 \rightarrow \boxed{4, 25}$

(c) $x_1, x_2 \bmod 3 = 0$
$x_1, x_2 \bmod 7 = 2$

From the table created in (a) we see that the smallest x for which $f(x) = (0, 2)$ is $x = 9$
$9 + (3 \cdot 7) = 30 \rightarrow \boxed{9, 30}$