HW 1

1. let n_i be pairwise coprime. Now suppose there there exist an x and y such that x, y are solutions to all congruences then x and y give the same remainder when divided by n_i i.e $x-y$ is a multiple of each n_i. This would mean that the product of n_i, lets call it N, $N \mid x-y$ therefore x and y are congruent mod N.
Now take the map $x \bmod N \to (x \bmod n_1, \dots x \bmod n_k)$. Since this map is injective and surjective, then there exist a solution.

2. $\frac{1}{2}+\frac{1}{3}+\frac{1}{5}=\frac{31}{30}$

$\frac{31}{30}=\text{reciprocal}(\lceil \frac{30}{31} \rceil)+(\frac{31}{30}-\text{reciprocal}(\lceil \frac{30}{31} \rceil))$

$=\frac{1}{1}+\frac{1}{30}$

I think $\frac{1}{2}+\frac{1}{3}+\frac{1}{5}$ will be better then $\frac{1}{1}+\frac{1}{30}$ in terms of pizza because spliting a pizza into 30 equally peices is hard and time consuming.

3a. $f(x) = (x \pmod 3, x \pmod 7)$

$f(1)=(1,1)$ $f(2)=(2,2)$ $f(3)=(0,3)$

$f(4)=(1,4)$ $f(5)=(2,5)$ $f(6)=(0,6)$

$f(7)=(1,0)$ $f(8)=(2,1)$ $f(9)=(0,2)$

$f(10)=(1,3)$ $f(11)=(2,4)$ $f(12)=(0,5)$

$f(13)=(1,6)$ $f(14)=(2,0)$ $f(15)=(0,1)$

$f(16)=(1,2)$ $f(17)=(2,3)$ $f(18)=(0,4)$

$f(19)=(1,5)$ $f(20)=(2,6)$

$20+21K$

20 and 41 are the smallest integers.
S.t $x \equiv 2 \pmod 3$ and $x \equiv 6 \pmod 7$

3b. $4+21K$ by table above since moduli are the same.

4 and 25 are the smallest integers
S.t $x \equiv 1 \pmod 3$ and $x \equiv 4 \pmod 7$

3c. $9+21K$ by table above.

9 and 30 are the smallest integers
S.t $x \equiv 0 \pmod 3$ and $x \equiv 2 \pmod 7$