

1) How the Chinese Remainder Method works

If both functions of x are divisible by n_1 and by n_2 and relatively prime, x is a multiple of n_1, n_2 . We create a table with $f(x) = (x \pmod{n_1}, x \pmod{n_2})$. This allows us to have a table of values where we find a_1, a_2, \dots, a_n and look at the table constructed by the given size (should be no bigger than $n_1 * n_2$) where $f(x) = (a_1, a_2)$

$$2) \frac{1}{2} + \frac{1}{3} + \frac{1}{5} = \frac{31}{30}$$

The inverse of $\frac{31}{30}$ is $\frac{30}{31}$ and the smallest integer larger than that fraction is 1 so you obtain

$$\begin{aligned} \frac{31}{30} &= 1 + EGF \left(\frac{31}{30} - 1 \right) \\ &= 1 + \frac{1}{30} \end{aligned}$$

In this way, each child gets 1 pie each, then $\frac{1}{30}$ of the last pie is split. This is compared to the previous breakdown of $\frac{31}{30} = \frac{1}{2} + \frac{1}{3} + \frac{1}{5}$ where each child gets $\frac{1}{2}, \frac{1}{3},$ and $\frac{1}{5}$ of a pie making the splitting of 30 pies easier and bigger pieces. The greedy algorithm is not a good way to divide the pizza as the last pie is cut so small, it makes it impractical to accomplish.

3)

a) $(2, \text{mod } 3), (6 \text{ mod } 7)$

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|-----------------|-----------------|-----------------|-----------------|-----------------|
| $f(1) = (1,1)$ | $f(2) = (2,2)$ | $f(3) = (0,3)$ | $f(4) = (1,4)$ | $f(5) = (2,5)$ |
| $f(6) = (0,6)$ | $f(7) = (1,0)$ | $f(8) = (2,1)$ | $f(9) = (0,2)$ | $f(10) = (1,3)$ |
| $f(11) = (2,4)$ | $f(12) = (0,5)$ | $f(13) = (1,6)$ | $f(14) = (2,0)$ | $f(15) = (0,1)$ |
| $f(16) = (1,2)$ | $f(17) = (2,3)$ | $f(18) = (0,4)$ | $f(19) = (1,5)$ | $f(20) = (2,6)$ |

$$x_2 = x_1 + 3 * 7$$

2 lowest integers are 20, 41

b) $(1, \text{mod } 3), (4, \text{mod } 7)$

Using the table above we see that 4 and 25 have this property

c) $(0, \text{mod } 3), (2, \text{mod } 7)$

Using the table above, we see that 9 and 30 have the properties described.