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(1.) Chinese Remainder Theorem

Def: If a, b are integers, then

$$[a,b] := \{n \in \mathbb{Z} \mid a \le n \le b\}$$

Theorem: If n_1 and n_2 are relatively prime ($\gcd(n_1, n_2) = 1$) then the mappings

$f: [0, n_1 n_2 - 1] \to [0, n_1 - 1] \times [0, n_2 - 1]$,

defined by: $f(x) := (x \pmod{n_1}, x \pmod{n_2})$ is one-to-one (a bijection)

Proof: If $f(x) = (0,0)$, then x is divisible by n_1 and n_2. Since n_1 and n_2 are relatively prime, it is a multiple of $n_1 n_2$ and since $0 \le x < n_1 n_2$, $x = 0$

So if $f(x_1) = f(x_2)$, then $f(x_1 - x_2) = 0$ and hence $x_1 - x_2 = 0$, so $x_1 = x_2$. So we have a bijective mapping.

Because $[0, n_1 n_2 - 1]$ has the same cardinality as $[0, n_1 - 1] \times [0, n_2 - 1]$ it is one-to-one it must be onto so it has an inverse.

(2.) $\frac{1}{2} + \frac{1}{3} + \frac{1}{5}$ into a unit fraction.

$2 \cdot 3 \cdot 5 = 30$, $\left[\frac{15}{30} + \frac{10}{30} + \frac{6}{30}\right] = \boxed{\frac{31}{30}}$

Greedy Algorithm

$1\frac{1}{30}$ reciprocal $= \frac{30}{1}$, $\text{ceil}(30) = 30$, $E7(1 - \frac{1}{30}) = \frac{1}{2} + \frac{1}{3} + \frac{1}{8} + \frac{1}{30} + \frac{1}{120}$

Dividing pizzas is easier with the egyptian fraction $\frac{1}{2}, \frac{1}{3}, \frac{1}{5}$ because there is fewer computations to manage.

(3.) Find the smallest integers such that:

(a) It leaves remainder 2 when divided by 3

↳ remainder 6 when divided by 7

$a = x \cdot b + r \to \begin{bmatrix} x = 3k + 2 \\ x = 7k + 6 \end{bmatrix}$ $a \equiv b \pmod{n} \to 2 = x \pmod 3$, $f(x) = (x \bmod 3, x \pmod 7)$

$\to 6 = x \pmod 7$ $f(0) = (0,0)$, $f(1) = (1,1)$

$x = 20, 41$ have this property. $\boxed{\begin{matrix} 20 = (3 \cdot 5) + 2, & 20 = 7 \cdot 2 + 6 \\ 41 = 3 \cdot 13 + 2, & 41 = 7 \cdot 5 + 6 \end{matrix}}$

$x = \frac{9}{3}, 2$

(5.) leaves r 1 when divided by 3, $\frac{x}{3} = k\,r\,1 \to x \bmod 3 = 1$

leaves r 4 when divided by 7, $\frac{x}{7} = k\,r\,4 \to x \bmod 7 = 4$

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$$\left.\begin{aligned} x \pmod 3 &= 2 \\ x \pmod 7 &= 4 \end{aligned}\right\} \quad f(x) := (x \pmod 3,\ x \pmod 7)$$

$f(0)=(0,0)$, $f(1)=(1,1)$, $f(2)=(2,2)$, $f(3)=(0,4)$, $f(4)=(1,3)$

$f(5)=(2,5)$, $f(6)=(0,6)$, $f(7)=(1,0)$, $f(8)=(2,1)$, $f(9)=(0,2)$, $f(10)=(1,3)$

$\cdots \boxed{f(11)=(2,4)} \cdots f(17)=(2,3) \cdots f(25)=(1,4) \cdots \boxed{f(32)=(2,4)}$

81

79

$\boxed{x=11,32}$

(c) Leaves remainder $\boxed{0}$ when divided by 3, $\frac{x}{3}=c+0$, $f(x):((x \bmod 3),\ x \pmod 7))$

Leaves remainder $\boxed{2}$ when divided by 7, $\frac{x}{7}=c+2$,

$f(0)=(0,0)$, $f(1,1)$

$$\left.\begin{aligned} f(9)&=(0,2) \\ f(30)&=(0,2) \end{aligned}\right\} \quad \boxed{x=9,30}$$