Homework for Dr. Z.'s MathHistory for Lecture 1

1. For every integers a and b, there exist integer $a \le n \le b$,

$f(x) := (x \pmod{n_1}, x \pmod{n_2})$

Add mode definition here. which means, x is the original number divided by n_1, x is the remainder. by listing all the posible remainder we could easily find out the smallest integer met the requirement.

2. $\frac{1}{2} + \frac{1}{3} + \frac{1}{5} = \frac{31}{30}$

$\frac{1}{x} = \frac{30}{31}$ ceil $(\frac{1}{x}) = \frac{1}{2}, \frac{1}{3}, \frac{1}{5}$

The first way is easier, because for cutting a cake, $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{5}$ is easier to divide than $\frac{30}{31}$

3 (a) Let's find the smallest integer x that

$x \bmod 3 \equiv 2$ $\quad x \bmod 7 \equiv 6$

1(2, 6) 2(1, 5) 3(0, 4) 4(1, 3) 5(2, 2) 6(0, 1)

7(1, 0), 8(2, 1) 9(0, 2) 10(1, 3) 11 (2, 4) 12(0, 5)

13(1, 6) 14(2, 0) 15(0, 1) 16(1, 2) 17(2, 3) 18(0, 4)

19(1, 5) 20(2, 6)

The smallest integers are 1 and 20

(b) Let's find the smallest two integer x that

$x \bmod 3 \equiv 1$ $\quad x \bmod 7 \equiv 4$

$n = 3k_1 + 1$ $\quad n = 7k_2 + 4$

$3k_1 + 1 = 7k_2 + 4$

$7k_2 - 3k_1 = 3$

$k_1 = 8, \; k_2 = 3$

$k_1 = 15 \; k_2 = 6$

The smallest integers are 25 and 46

(C) $x \equiv 0 \pmod 3$ $\quad x \equiv 2 \pmod 7$

$n = 3k_1$ $\quad n = 7k_2 + 2$

$k_1 = 10$ $\quad k_2 = 4$

$k_1 = 17$ $\quad k_2 = 7$

the smallest integers are 30 and 51