1. Suppose $f(x) = (0, 0)$. Then $x \mod n_1$ and $x \mod n_2$ both equal zero, meaning that n_1 divides x and n_2 divides x. Because n_1 and n_2 are relatively prime, they have entirely distinct factorizations; in other words, a factorization of x, which both n_1 and n_2 divide, must include the product of n_1 and n_2. In simpler terms, x is a multiple of $n_1 n_2$.

   It's known by the domain of the mapping that $0 <= x < n_1 n_2$. So x is known to be a nonnegative multiple of $n_1 n_2$ that is less than $n_1 n_2$. This is only possible when $x = 0$.

   To prove by contrapositive, assume that $f(x_1) = f(x_2)$. Then $x_1 \mod n_1 = x_2 \mod n_1$ and $x_1 \mod n_2 = x_2 \mod n_2$. So $(x_2 - x_1) \mod n_1 = 0$ and $(x_2 - x_1) \mod n_2 = 0$. Therefore, $f(x_2 - x_1) = (0, 0)$, which is known to only be possible when the term passed to the function is 0. So $x_2 - x_1 = 0$, or $x_2 = x_1$. Thus the function is one-to-one.

   The domain of the function has $n_1 n_2$ elements and the range has $n_1 \times n_2$ elements, or also $n_1 n_2$ elements. Because the domain and codomain have the same cardinality, and no elements of the codomain can be mapped to by more than one element of the domain by the definition of one-to-one, (and no element of the domain can map to more than one element of the codomain by the definition of function,) the function must be onto as well as one-to-one. Thus by definition the function is bijective.

2. $$\frac{1}{2} + \frac{1}{3} + \frac{1}{5} = \frac{31}{30}$$

   $$\left\lceil \frac{30}{31} \right\rceil = 1$$
   $$\frac{1}{1} + \frac{31}{30} - \frac{1}{1}$$
   $$= \frac{1}{1} + \frac{1}{30}$$

   In the given way, 31 pizzas are divided amongst 30 people by giving each 1/2 of one pizza, 1/3 of another, and 1/5 of another. This is a much easier way to divide pizza than by dividing them per the result of the greedy algorithm, which would have each person get 1 pizza as well as 1/30 of another. While people hypothetically get the same amount of pizza with either method, in actuality it is near impossible to divide a pizza into 30 slices, and much more feasible to divide one into halves, thirds, or fifths.

3. (a) 20, 41
   (b) 25, 46
   (c) 9, 30