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History of Math
Homework 0

(1) Express one hundred (base ten) in

(a) base two

$100 = 1(2^6) + 1(2^5) + 0(2^4) + 0(2^3) + 1(2^2) + 0(2^1) + 0(2^0)$

$100 = (1100100)_2$

(b) base three

$100 = 1(3^4) + 0(3^3) + 2(3^2) + 0(3^1) + 1(3^0)$

$100 = (10201)_3$

(c) base four

$100 = 1(4^3) + 2(4^2) + 1(4^1) + 0(4^0)$

$100 = (1210)_4$

(d) base 5

$100 = 4(5^2) + 0(5^1) + 0(5^0)$

$100 = (400)_5$

(e) base six

$100 = 2(6^2) + 4(6^1) + 4(6^0)$

$100 = (244)_6$

(f) base seven

$100 = 2(7^2) + 0(7^1) + 2(7^0)$

$100 = (202)_7$

(g) base eight.

$100 = 1(8^2) + 4(8^1) + 4(8^0)$

$100 = (144)_8$

(h) base nine

$100 = 1(9^2) + 1(9^1) + 0(9^0)$

$100 = (110)_9$

(i) base 10

$100 = 1(10^2) + 0(10^1) + 0(10^0)$

$100 = (100)_{10}$

(j) base 11

$100 = 9(11^1) + 1(11^0)$

$100 = (91)_{11}$

(k) base 12

$100 = 8(12^1) + 4(11^0)$

$100 = (84)_{12}$

(2) Use the Todd-Zeilberger algorithm (see link in the webpage of class) to multiply 101 by 97.

$101 = (1, 0, 1)$

$97 = 100 - 3 = (1, 0, -3)$

$$\begin{array}{r} 1\ 0\ 1 \\ \times\ 1\ 0\ {-3} \\ \hline -3\ 0\ -3 \\ +1\ 0\ 1\ 0\ 0 \\ \hline 1, 0, -2, 0, -3 \end{array} \rightarrow \boxed{9797}$$

(3) Use the base three analog of the Todd-Zeiberger algorithm to multiply $26_{10} \times 80_{10}$ by first converting 26_{10} and 80_{10}

$26_{10} = 1(3^3) + 0(3^2) + 0(3^1) - 1(3^0) = (1, 0, 0, -1)$

$80_{10} = 1(3^4) + 0(3^3) + 0(3^2) + 0 - 1(3^0) = (1, 0, 0, 0, -1)$

$$
\begin{array}{r}
1, 0, 0, 0, -1 \\
\times \quad 1 \; 0 \; 0 \; -1 \\
\hline
-1 \; 0 \; 0 \; 0 \; 1 \\
+1 \; 0 \; 0 \; 0 \; -1 \; 0 \; 0 \; 0 \\
\hline
1 \; 0 \; 0 \; -1 \; -1 \; 0 \; 0 \; 1
\end{array}
$$

$1(3^7) - 1(3^4) - 1(3^3) + 1(3^0) = \boxed{2080}$

(4) Construct a base three chart analog to the guessing game given in class that includes all the numbers 26.

$1_{10} = 1_3$
$2_{10} = 2_3$
$3_{10} = 10_3$
$4_{10} = 11_3$
$5_{10} = 12_3$
$6_{10} = 20_3$
$7_{10} = 21_3$
$8_{10} = 22_3$
$9_{10} = 100_3$
$10_{10} = 101_3$
$11_{10} = 102_3$
$12_{10} = 110_3$
$13_{10} = 111_3$
$14_{10} = 112_3$
$15_{10} = 120_3$
$16_{10} = 121_3$
$17_{10} = 122_3$
$18_{10} = 200_3$
$19_{10} = 201_3$
$20_{10} = 202_3$
$21_{10} = 210_3$
$22_{10} = 211_3$
$23_{10} = 212_3$
$24_{10} = 220_3$
$25_{10} = 221_3$
$26_{10} = 222_3$

1 4 7	2 5 8	3 4 5
10 13 16	11 14 17	12 13 14
19 22 25	20 23 26	21 22 23
6 7 8	9 10 11	18 19 20
15 16 17	12 13 14	21 22 23
24 25 26	15 16 17	24 25 26