

① Summarize Chapter I :

Chapter I of the textbook takes readers back to the most basic foundation of mathematics. The author discusses the first concept of the number system, which dates back to the Old Stone Age. During the times before this, the general population had minimal understanding of numerical values. Eventually, when hunting and gathering transformed into agriculture and production. This was the start of a whole new era of development, including crafts such as pottery, carpentry, and weaving. Numerical terms for new developments only slowly came into use. First came qualitative conceptions, which slowly became quantitative distinctions. Different bases (such as base 5, and base 10) were invented, and then arithmetic operations within these bases. Then, civilization progressed to measure the size and contents of various objects, and these measurements eventually became more complicated. This was just the beginning of mathematics. As we can see, it grew quickly and is constantly used in today's world.

② Express 100_{10} in

a) base two:

$$1(2^6) + 1(2^5) + 0(2^4) + 0(2^3) + 1(2^2) + 0(2^1) + 0(2^0) \\ = 1100100_2$$

b) base three:

$$1(3^4) + 0(3^3) + 2(3^2) + 0(3^1) + 1(3^0) \\ = 10201_3$$

c) base four:

$$1(4^3) + 2(4^2) + 1(4^1) + 0(4^0) \\ = 1210_4$$

d) base five:

$$4(5^2) + 0(5^1) + 0(5^0) \\ = 400_5$$

e) base six:

$$2(6^2) + 4(6^1) + 4(6^0) \\ = 244_6$$

f) base seven:

$$2(7^2) + 0(7^1) + 2(7^0) \\ = 202_7$$

g) base eight:

$$1(8^2) + 4(8^1) + 4(8^0) \\ = 144_8$$

h) base nine:

$$1(9^2) + 2(9^1) + 1(9^0) \\ = 121_9$$

i) base ten:

$$1(10^2) + 0(10^1) + 0(10^0) \\ = 100_{10}$$

j) base eleven

$$9(11^2) + 1(11^0) \\ 91_{11}$$

k) base twelve

$$8(12^2) + 4(12^0) \\ = 84_{12}$$

$$\textcircled{2} \quad 101 = 100 + 1 = 2(10^2) + 0(10^1) + 1(10^0) \\ = (1, 0, 1)$$

$$97 = 100 - 3 = 1(10^2) - 1(10^1) + 7(10^0) \\ = (1, -1, 7)$$

$$101 \cdot 97 = \begin{array}{r} 1 \ 0 \ 1 \\ \times 1 \ -1 \ 7 \\ \hline 7 \ 0 \ 7 \\ -1 \ 0 \ -1 \\ \hline 1 \ 0 \ 1 \\ \hline 1 \ -1 \ 8 \ -1 \ 7 \end{array} \quad \left. \begin{array}{l} (1, -1, 8, -1, 7) \\ = (9, 8, -1, 7) \\ = (9, 7, 9, 7) \\ = \boxed{9797} \end{array} \right\}$$

$$\textcircled{3} \quad 26_{10} = 2(3^2) + 2(3^1) + 2(3^0) \\ = 222_3$$

$$80_{10} = 2(3^3) + 2(3^2) + 2(3^1) + 2(3^0) \\ = 2\bar{2}\bar{2}\bar{2}_3$$

$$\begin{array}{r} 2222 \\ \times 222 \\ \hline 12221 \\ 12221 \\ 12221 \\ \hline 2212001 \end{array} \quad \left. \begin{array}{l} (2, 2, 1, 2, 0, 0, 1) \\ = \boxed{2212001_3} \end{array} \right\}$$

(4) 1 = 1
 2 = 2
 3 = 10
 4 = 11
 5 = 12
 6 = 20
 7 = 21
 8 = 22
 9 = 100
 10 = 101
 11 = 102
 12 = 110
 13 = 111
 14 = 112
 15 = 120
 16 = 121
 17 = 122
 18 = 200
 19 = 201
 20 = 202
 21 = 210
 22 = 211
 23 = 212
 24 = 220
 25 = 221
 26 = 222

1 4 7
 10 13 16
 19 22 25

2 5 8
 11 14 17
 20 23 26

3 4 5
 12 13 14
 21 22 23

6 7 8
 15 16 17
 24 25 26

9 10 11
 12 13 14
 15 16 17

18 19 20
 21 22 23
 24 25 26