

Chapter 1: Counting became more developed as society developed more. Seeing forms similar to the tally system in caves negates the idea that base 5 or 10, due to the number of fingers, was the earliest known system of counting. Base 10 came later on, but the idea that larger numbers are built off smaller numbers has been indicated in some of the language of ancient civilization. Perhaps, this is how addition came about. Measurement tended to be based on human body parts such as feet or arms, which made it not so precise. Intricacy could be seen in pottery and design, pointing to possible early development of geometry.

$$1. a) 100 = 64 + 32 + 4 \rightarrow 1 \cdot 2^6 + 1 \cdot 2^5 + 0 \cdot 2^4 + 0 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 0 \cdot 2^0$$

$$1100100$$

$$b) 100 = 81 + 9 + 9 + 1 \rightarrow 1 \cdot 3^4 + 0 \cdot 3^3 + 2 \cdot 3^2 + 0 \cdot 3^1 + 1 \cdot 3^0 = 10201$$

$$c) 100 = 64 + 16 + 16 + 4 \rightarrow 1 \cdot 4^3 + 2 \cdot 4^2 + 1 \cdot 4^1 + 0 \cdot 4^0 = 1210$$

$$d) 100 = 25 + 25 + 25 + 25 = 4 \cdot 5^2 + 0 \cdot 5^1 + 0 \cdot 5^0 = 4$$

$$e) 100 = 36 + 36 + 6 + 6 + 6 + 6 + 1 + 1 + 1 + 1 = 2 \cdot 6^2 + 4 \cdot 6^1 + 4 \cdot 6^0 = 244$$

$$f) 100 = 49 + 49 + 1 + 1 = 2 \cdot 7^2 + 0 \cdot 7^1 + 2 \cdot 7^0 = 202$$

$$g) 100 = 64 + 8 + 8 + 8 + 8 + 1 + 1 + 1 + 1 = 1 \cdot 8^2 + 4 \cdot 8^1 + 4 \cdot 8^0 = 144$$

$$h) 100 = 81 + 9 + 9 + 1 = 1 \cdot 9^2 + 2 \cdot 9^1 + 1 \cdot 9^0 = 121$$

$$i) 100 = 1 \cdot 10^2 + 0 \cdot 10^1 + 0 \cdot 10^0 = 100$$

$$j) 100 = 9 \cdot 11^1 + 1 \cdot 11^0 = 91$$

$$k) 100 = 8 \cdot 12^1 + 4 \cdot 12^0 = 84$$

$$2. 101 \times 97$$

$$101 = 100 + 1 = 1 \cdot 10^2 + 1 \cdot 10^0 = (1, 0, 1)$$

$$97 = 100 - 3 = 1 \cdot 10^2 - 3 \cdot 10^0 = (1, 0, -3)$$

$$10-3$$

$$\times 10-1$$

$$1^2 0 -3$$

$$0 0 0$$

$$10-3$$

$$10-2 0 -3$$

$$10000 - 200 - 3 = 9797$$

3.  $26_{10} \times 80_{10}$

$26 = 2 \times 10^1 + 6 \times 10^0 = 26$      $80 = 8 \times 10^1 + 0 \times 10^0 = 80$

Base 3:  $26 = 2 \times 3^2 + 2 \times 3^1 + 2 \times 3^0 = (2, 2, 2)_3$

$80 = 1 \times 3^4 - 1 \times 3^0 = (1, 0, 0, 0, 1)_3$

1000-1  
 x 222  
 2000-2  
 2000-2  
 2000-2  
 2220-2-2-2

$2 \times 3^6 + 2 \times 3^5 + 2 \times 3^4 + 0 \times 3^3 - 2 \times 3^2 - 2 \times 3^1 - 2 \times 3^0$

$1458 + 486 + 162 - 18 - 6 - 2 = 2080$

4.  $1 = 1 \cdot 3^0$ ;  $2 = 2 \cdot 3^0 = 1 \cdot 3^1 - 1 \cdot 3^0$ ;  $3 = 3^0$ ;  $4 = 3^1 + 3^0 = 3^2 - 3^1 - 2 \cdot 3^0$

$5 = 1 \cdot 3^1 + 2 \cdot 3^0 = 1 \cdot 3^2 - 1 \cdot 3^1 - 1 \cdot 3^0$ ;  $6 = 2 \cdot 3^1 + 0 \cdot 3^0 = 1 \cdot 3^2 - 1 \cdot 3^1$ ;

$7 = 2 \cdot 3^1 + 1 \cdot 3^0 = 1 \cdot 3^2 - 2 \cdot 3^0$ ;  $8 = 2 \cdot 3^1 + 2 \cdot 3^0 = 1 \cdot 3^2 - 1 \cdot 3^0$ ;

$9 = 1 \cdot 3^2 = 3 \cdot 3^1$ ;  $10 = 1 \cdot 3^2 + 1 \cdot 3^0 = 3 \cdot 3^1 + 1 \cdot 3^0$ ;  $11 = 1 \cdot 3^2 + 2 \cdot 3^0 = 3 \cdot 3^1 + 2 \cdot 3^0$ ;

$12 = 1 \cdot 3^2 + 1 \cdot 3^1 = 4 \cdot 3^1$ ;  $13 = 1 \cdot 3^2 + 1 \cdot 3^1 + 1 \cdot 3^0 = 4 \cdot 3^1 + 1 \cdot 3^0$ ;

$14 = 1 \cdot 3^2 + 1 \cdot 3^1 + 2 \cdot 3^0 = 5 \cdot 3^1 - 1 \cdot 3^0$ ;  $15 = 1 \cdot 3^2 + 2 \cdot 3^1$ ;  $16 = 1 \cdot 3^2 + 2 \cdot 3^1 + 1 \cdot 3^0$ ;

$17 = 2 \cdot 3^2 - 1 \cdot 3^0 = 1 \cdot 3^2 + 2 \cdot 3^1 + 2 \cdot 3^0$ ;  $18 = 2 \cdot 3^2$ ;  $19 = 2 \cdot 3^2 + 1 \cdot 3^0$ ;

$20 = 2 \cdot 3^2 + 2 \cdot 3^0$ ;  $21 = 2 \cdot 3^2 + 1 \cdot 3^1$ ;  $22 = 2 \cdot 3^2 + 1 \cdot 3^1 + 1 \cdot 3^0$ ;  $23 = 2 \cdot 3^2 + 2 \cdot 3^1 - 1 \cdot 3^0$ ;

$24 = 2 \cdot 3^2 + 2 \cdot 3^1 = 1 \cdot 3^3 - 1 \cdot 3^1$ ;  $25 = 1 \cdot 3^3 - 2 \cdot 3^0$ ;  $26 = 1 \cdot 3^3 - 1 \cdot 3^0$

1 2 4 5 7	3 4 5 6	9 7 8 10
8 10 11 13 14	7 8 12 13	11 12 13 14
16 17 19 20 22	14 15 16 21	15 16 17 18 23
23 25 26	22 23 24	19 20 21 22

27 24 25  
 26