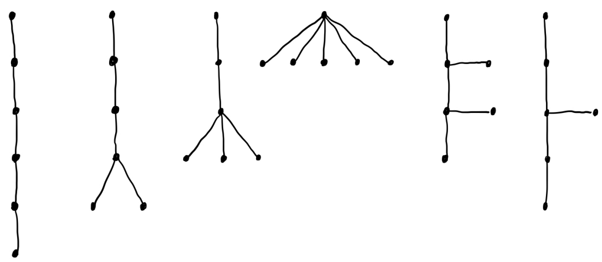
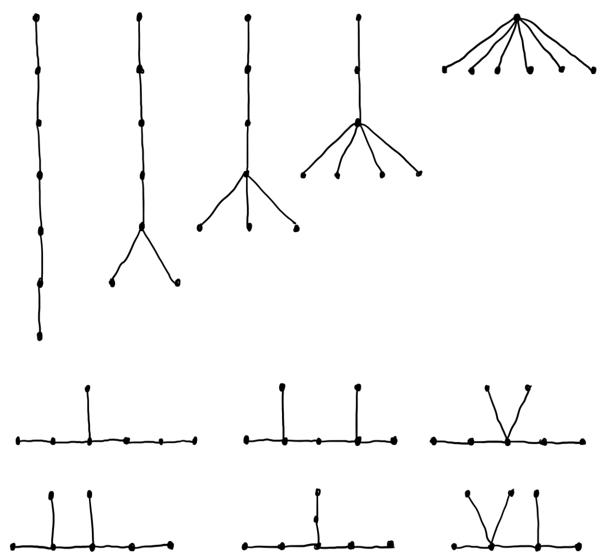
Holen Yee
Graph Theory Homework #9

9.1)
Trees in Fig. 2.9: 1, 2, 3, 5, 6, 11, 12, 13

9.2)
Trees on six vertices:



Trees on seven vertices:



9.3)

(i) Let T be an arbitrary tree. Choose an arbitrary vertex of T, which we will call v. Since T is a tree, for every vertex u, there is a unique path from v to u. Partition the set of vertices of T, V, into two sets A and B, defined by:

$A = \{u \in V : \text{length of path from } v \text{ to } u \text{ is even}\}$

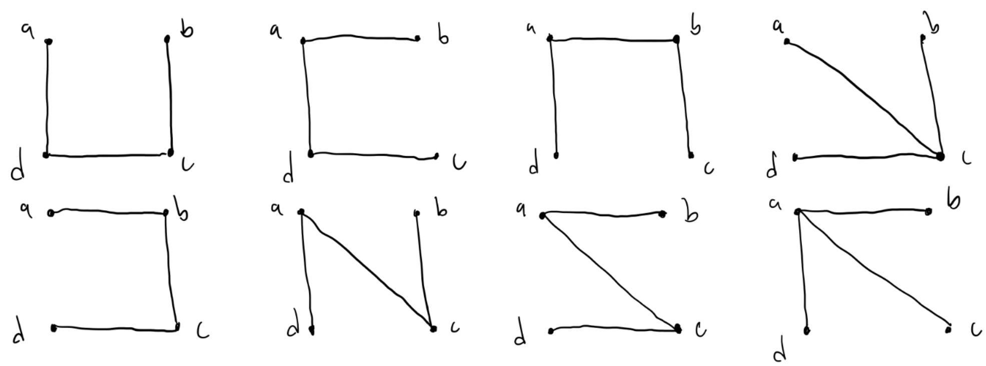
$B = \{u \in V : \text{length of path from } v \text{ to } u \text{ is odd}\}$

We claim that for any $x, y \in A$, there is no edge connecting x and y and that for any $x, y \in B$, there is no edge connecting x and y.

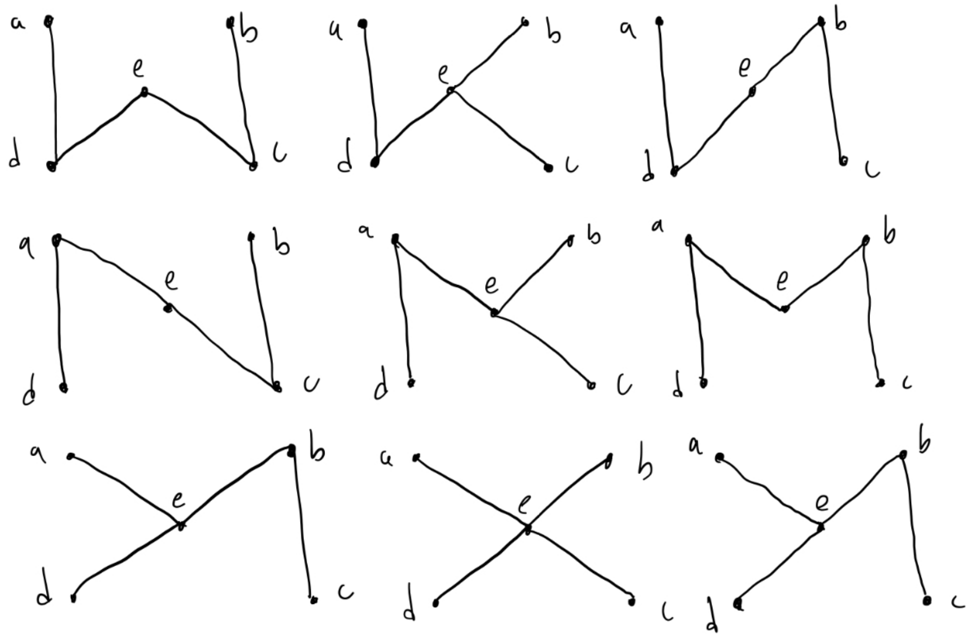
Suppose for contradiction that there exist $x, y \in A$ such that there is an edge connecting x and y. Since $x, y \in A$, the path from v to x and the path from v to y are of even length. We can construct a new path from v to y of odd length by following the path from v to x, then using the edge between x and y. This contradicts the property of trees that the path from any vertex to any other vertex is unique. So, there are no edges between the vertices in A and no edges between the vertices in B. So, T is bipartite. Since T was arbitrary, all trees are bipartite.

(ii) $K_{r,s}$, where $r = 1$ or $s = 1$

9.4)



9.5)



9.6)
Cycles: $a \to b \to c \to d \to e \to a$, $a \to b \to c \to a$, $a \to b \to c \to d \to a$, $c \to d \to c$
Cutsets: $\{ab, ac, ad, ae\}$, $\{ac, ad, ae, bc\}$, $\{ad, ae, cd, cd\}$, $\{ae, de\}$

9.7)

(i) $\gamma(K_5) = 6$
$\xi(K_5) = 4$

(ii) $\gamma(K_{3,3}) = 13$
$\xi(K_{3,3}) = 5$

(iii) $\gamma(W_5) = 4$
$\xi(W_5) = 4$

(iv) $\gamma(N_5) = 0$
$\xi(N_5) = 0$

(v) $\gamma(\text{The Petersen graph}) = 6$
$\xi(\text{The Petersen graph}) = 9$

9.8)

(i) That edge is a bridge.

(ii) That edge is a loop.