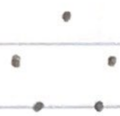
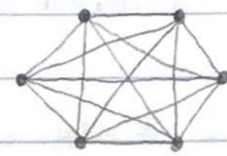
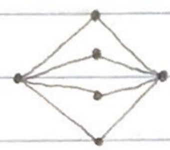
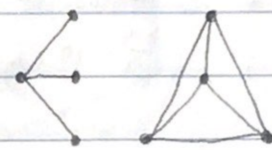
Homework 3

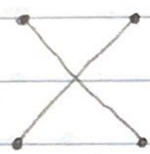
3.1)

(i) 

(ii) 

(iii) 

(iv) 

(v) 

3.2)

(i) K_n has $\frac{n(n-1)}{2}$ edges.

$\frac{10(10-1)}{2} = \frac{90}{2} = 45$

K_{10} has 45 edges.

(ii) $K_{r,s}$ has rs edges.

$(5)(7) = 35$

$K_{5,7}$ has 35 edges.

(iii) Q_k has $k2^{k-1}$ edges.

$4 \cdot 2^{4-1} = 4 \cdot 8 = 32$

Q_4 has 32 edges.

(iv) 7 vertices of degree 3.

1 vertex of degree 7.

$\frac{7 \cdot 3 + 7}{2} = \frac{21+7}{2} = \frac{28}{2} = 14$

W_8 has 14 edges.

(v) Petersen graph is regular of degree 3 with 10 vertices.

$\frac{3 \cdot 10}{2} = \frac{30}{2} = 15$

The Petersen graph has 15 edges.

3.3) Tetrahedron: 4 vertices by counting.

Regular of degree 3.

$\frac{4 \cdot 3}{2} = 6$ edges.

Octahedron: 6 vertices by counting.

Regular of degree 4.

$\frac{6 \cdot 4}{2} = 12$ edges.

Cube: 8 vertices by counting.

Regular of degree 3.

$\frac{8 \cdot 3}{2} = 12$ edges.

Icosahedron: 12 vertices by counting.

Regular of degree 5.

$\frac{12 \cdot 5}{2} = 30$ edges.

Dodecahedron: 20 vertices by counting.

Regular of degree 3.

$\frac{20 \cdot 3}{2} = 30$ edges.

3.4) Regular: 1, 2, 4, 8, 10, 18, 31

Bipartite: 2, 3, 5, 6, 8, 11, 12, 13, 17, 23

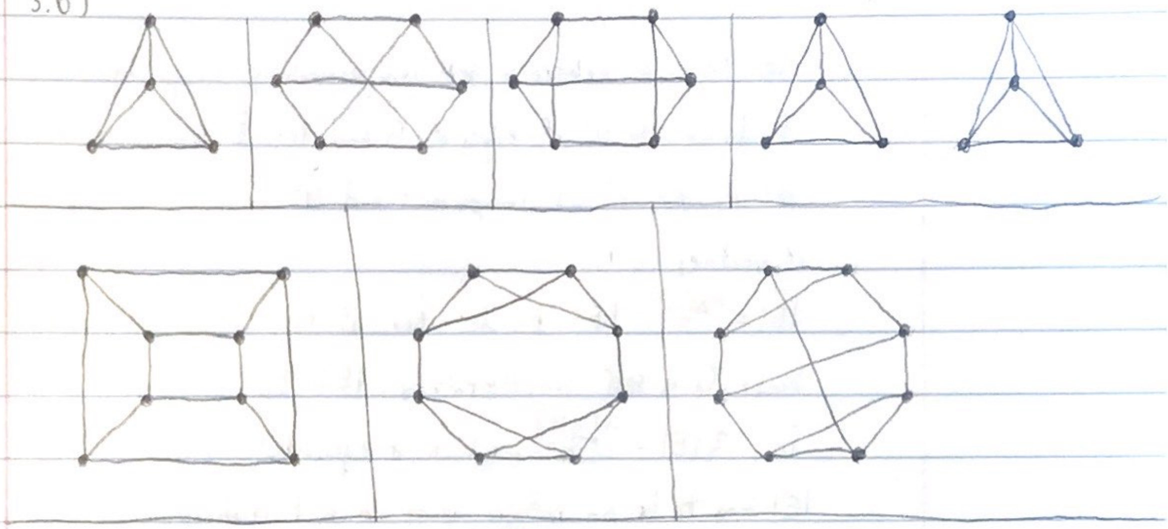
3.5)

(i) $K_{3,5}$ (ii) Cube (iii) K_4

(iv) $\frac{11 \cdot 3}{2} = \frac{33}{2}$. Not integer, impossible # of edges. Does not exist.

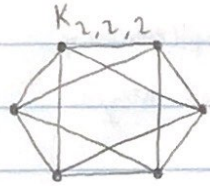
(v) Octahedron

3.6)

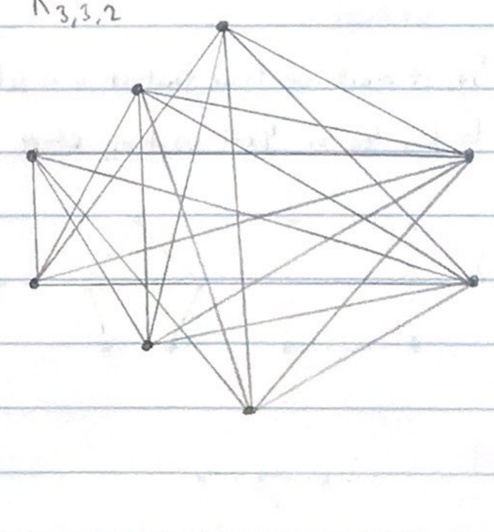


3.7)

$K_{2,2,2}$



$K_{3,3,2}$



$K_{3,4,5}$

$(3)(4)+(3)(5)+(4)(5) = 12+15+20 = 47$ edges.

3.8)

(i) Let G be an arbitrary self-complementary graph. Let E denote the set of edges of G and let $\bar{E}$ denote the set of edges of G's complement and let n denote the number of vertices in G.

$|\bar{E}| = \frac{n(n-1)}{2} - |E|$ by definition of the complement.

Since G is self-complementary, $|E| = |\bar{E}|$.

So, $2|E| = \frac{n(n-1)}{2}$, which is equivalent to $|E| = \frac{n(n-1)}{4}$.

$|E|$ has to be an integer, so n or $n-1$ is divisible by 4.

In the case that n is divisible by 4:

$\frac{n}{4} = k$ for some integer k.

$\frac{n}{4} = k \Rightarrow n = 4k$, so G has $4k$ vertices.

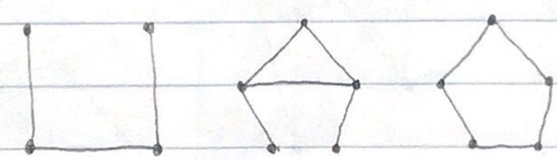
In the case that $n-1$ is divisible by 4:

$\frac{n-1}{4} = k$ for some integer k.

$\frac{n-1}{4} = k \Rightarrow n-1 = 4k \Rightarrow n = 4k+1$, so G has $4k+1$ vertices.

So, it must be true that if G is self-complementary, G has $4k$ or $4k+1$ vertices, where k is an integer.

(ii)



(iii)