Homework 2

2.2)

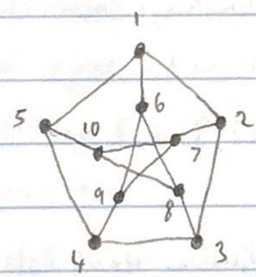
(i)

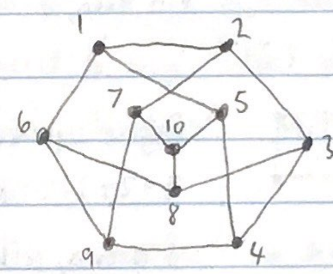
(ii)

(iii)

2.3)

(i)





(ii) In the graph to the left, every vertex of degree 2 is adjacent to two vertices of degree 3. For the two graphs to be isomorphic, this must also be true of the graph on the right. However, the vertices of the right graph are only adjacent to one vertex of degree 3. So, the two graphs are not isomorphic.

2.4)

(i) True

(ii) False

2.7)

Degree sequence: 1, 1, 1, 3

$1+1+1+3=6$

6 is even, so handshaking lemma holds.

Degree sequence: 1, 1, 2, 2

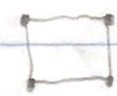
$1+1+2+2=6$

6 is even, so handshaking lemma holds.

Degree sequence: 1, 2, 2, 3

$1+2+2+3=8$

8 is even, so handshaking lemma holds.



Degree sequence: 2, 2, 2, 2

$2+2+2+2=8$

8 is even, so handshaking lemma holds.

Degree sequence: 2, 2, 3, 3

$2+2+3+3=10$

10 is even, so handshaking lemma holds.

Degree sequence: 3, 3, 3, 3

$3+3+3+3=12$

12 is even, so handshaking lemma holds.

2.9) Suppose for contradiction that G has no vertices whose degrees match.

Let n denote the number of vertices in G.

Since G is simple, a vertex of G can only have a degree of at most $n-1$.

In other words, the set of possible degrees for vertices of G is $\{0, 1, \dots, n-1\}$.

Since there are n possible degrees, n vertices in G, and we are assuming no two vertices have the same degree, the degree sequence of G must be $0, 1, \dots, n-1$.

However, it is a contradiction for G to have a vertex of degree 0 (not adjacent to any vertex) and a vertex of degree $n-1$ (adjacent to all other vertices) at the same time.

Since assuming that G has no vertices whose degrees match leads to a contradiction, it must be the case that G contains at least two vertices of the same degree.