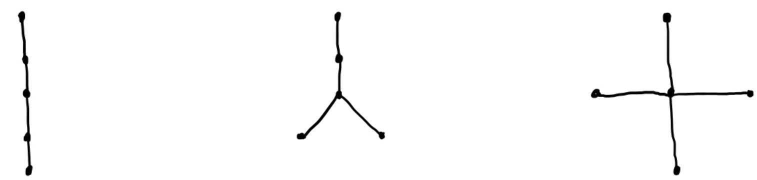
Holen Yee
Graph Theory Homework #10

10.1)

3 unlabeled trees:



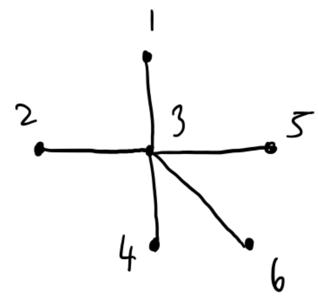
$\frac{5!}{2} = 60$ labelings $\qquad \frac{5!}{2!} = 60$ labelings $\qquad$ 5 labelings

$60 + 60 + 5 = 125$ labeled trees on 5 vertices.

10.2)

(i) The labeled tree corresponding to (1, 2, 3, 4):

The labeled tree corresponding to (3, 3, 3, 3):



(ii) The sequences corresponding to the trees in Figure 10.6 are (4, 4, 4, 1) and (4, 2, 2, 4).

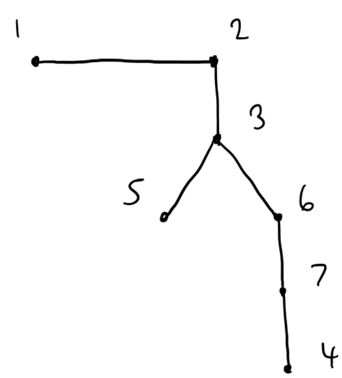
10.3)

(i) By Cayley's Theorem, there are $(n-1)^{n-3}$ trees on $n-1$ vertices. For each tree, we can form $n-1$ new trees by attaching our chosen vertex to each of the $n-1$ vertices as an end vertex. So, there are $(n-1)^{n-3}(n-1) = (n-1)^{n-2}$ trees on n vertices in which a given vertex is an end-vertex.

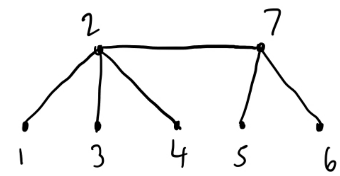
(ii) The probability that a given vertex is an end vertex is given by $\frac{(n-1)^{n-2}}{n^{n-2}} = (\frac{n-1}{n})^{n-2} = (\frac{n-1}{n})^{n}(\frac{n-1}{n})^{-2} = (1-\frac{1}{n})^{n}(1-\frac{1}{n})^{-2}$
As $n \to \infty$, $(1-\frac{1}{n})^{n}(1-\frac{1}{n})^{-2} \to (e^{-1})(1) = e^{-1}$, since $e^{-1} = \lim_{n\to\infty}(1-\frac{1}{n})^{n}$.
So, when n is large, this probability approximates e^{-1}.

Additional Problem 1:
The endofunctions corresponding to the doubly-rooted trees in Figure 10.6 where the first root is 1 and the second root is 2 are 144241 and 142224.

Additional Problem 2:
The doubly rooted tree corresponding to 1227336:



The doubly rooted tree corresponding to 2222777:


2
7
1
3
4
5
6