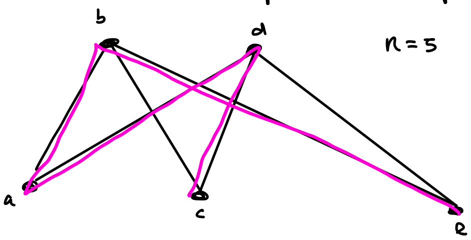
Section 7 : 7.1, 7.2, 7.3, 7.4, 7.5, 7.6, 7.7

7.1) Which of the following graphs are Hamiltonian? Semi-Hamiltonian?

(i) The complete graph K_5 : Hamiltonian

(ii) The complete bipartite graph $K_{2,3}$



$n=5$

$\deg(b)+\deg(d)=6$

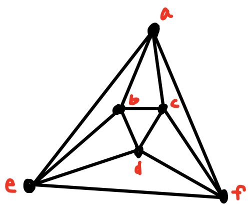
$\deg(a)+\deg(c)=4$ ×

NON HAMILTONIAN!

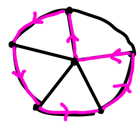
SEMI-HAMILTONIAN (See pink path)

(iii) The graph of the octohedron

$n=6$ Since every vertex has $\deg=4$, any two vertices' degrees added will be $8 \geq 6$, so HAMILTONIAN



(iv) The wheel W_6



Hamiltonian

(v) The 4-cube Q_4 Hamiltonian

7.2) In the table of Fig 2.9, locate all Hamiltonian and Semi-Hamiltonian graphs.

Hamiltonian: 1, 4, 8, 9, 10, 18, 22, 26, 27, 28, 29, 30, 31

Semi-Hamiltonian: 2, 3, 6, 7, 13, 15, 16, 17, 19, 20, 23, 24, 25

7.3)

(i) For which values of n is K_n hamiltonian?

When $n>2$.

(ii) Which complete Bipartite graphs are hamiltonian?

$K_{s,t}$ is hamiltonian if $s,t>1$ and $s=t$.

(iii) Which Platonic graphs are hamiltonian?

They are all Hamiltonian.

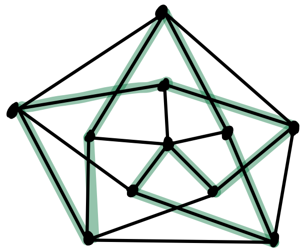
(iv) For which values of n is the wheel W_n hamiltonian.

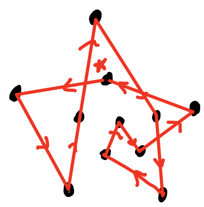
The Wheel W_n is Hamiltonian whenever it is a valid wheel ($n>2$).

(v) For which values of k is the k-cube hamiltonian?

When $k>1$.

7.4) Show that the Grötzsch graph is Hamiltonian





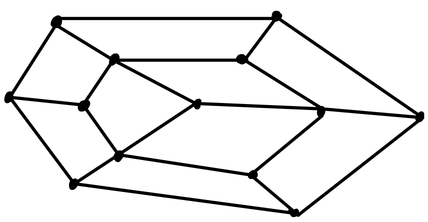
There is a hamiltonian cycle, therefore Hamiltonian.

7.5)

(i) Prove that if G is a bipartite graph with an odd number of vertices, then G is non-hamiltonian:

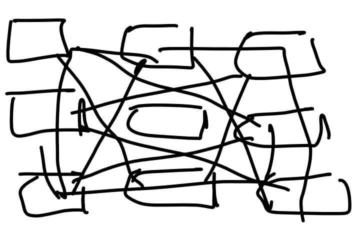
Say G is a bipartite graph with an odd number of vertices. Then, for K_{st}, $s \neq t$ since if it was, K would have an even number of vertices (if $s=t$, odd + odd = even or even + even = even). Since $s \neq t$, and all vertices in s, t are non-adjacent, we must jump from one set to another in the creation of our path (no other way) and we will end up in the larger set either with unvisited vertices in the larger set (we can't use the smaller set's vertices anymore since all of our vertices have been visited) ~~or~~ no ability to go back to the vertex we started on since we're in the disjoint set that includes the starting vertex.

(ii) Deduce that 7.7 is non-Hamiltonian:



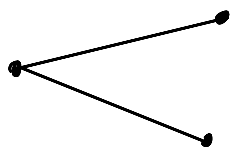
Since a hamiltonian cycle DNE, non-Hamiltonian!

(iii) Show if n is odd, not possible for a knight to visit all squares of an n×n chessboard exactly once by knight's moves & return to its starting point.



if n is odd, since a knight moves in "L" shape, which is two up/down or two left/right and one up/down or one left/right, there will be pieces never reached in the center as can be seen by this 3×3.

7.6) Show that $\deg(v) \geq n/2$ can't be replaced with $\deg(v) \geq \frac{(n-1)}{2}$



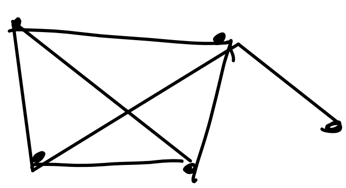
NO HAMILTONIAN CYCLE!

7.7)
(i) Let G be a graph with n vertizes and $[(n-1)(n-2)/2] + 2$ edges.

That's $\frac{n^2}{2} - \frac{3n}{2} + 3$ edges. Recall the complete graph with n vertices has $\frac{(n)(n-1)}{2} = \frac{n^2}{2} - \frac{n}{2}$ edges. So, the complete graph has $n-3$ more edges. also, the complete graph has, for any vertex, $\deg(v) = n-1$.

Say you take $n-3$ edges out of any vertex pair from K_n. Even if you take every edge out of a single pair, you still have $(n-1)+(n-1) - (n-3 = n+1$ degree in every pair of non-adj vertices (at least).

(ii)



$n = 5$

$\frac{(n-1)(n-2)}{2} + 1 = 7$