1

Graph Theory HW6

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SECTION 6: 6.1, 6.2, 6.3, 6.4, 6.5, 6.6

6.1) Which of the following graphs is Eulerian? Semi-Eulerian?

i) the complete graph K_5;

Ans: degree = $5-1=4$ (even). Therefore, K_5 is Eulerian.

ii) the complete bipartite graph $K_{2,3}$;

Ans: degree for $K_2 = 3$, degree for $K_3 = 2$

Therefore, $K_{2,3}$ is Semi-Eulerian

iii) the graph of the cube

Ans: degree = 3, (odd) Therefore, Cube is Not Eulerian or Semi-Eulerian (neither)

iv) the graph of the octahedron

Ans: degree = 4 (even). Therefore, Octahedron is Eulerian

v) the Petersen graph: 10 vertices, 15 edges

Ans: degree = 3 (odd). Therefore, Petersen Graph is neither

6.2) In the table of Fig 2.9, locate all the Eulerian and semi-Eulerian graphs.

Solution: i) Eulerian: 1, 4, 8, 18, 21, 25, 31

②

ii) Semi-Eulerian : 2, 3, 6, 7, 9, 13, 14, 16, 17, 19, 22, 23, 26, 28, 30.

6.3) (i) For which values of n is K_n Eulerian?

Answer:

K_n graphs have n−1 degrees. Therefore K_n is eulerian if n−1 is an even number

∴ K_n is Eulerian for values of n, where n−1 is an even number. e.g: K_3, K_5, K_9, K_{11}, ---

(ii) Which complete bipartite graphs are Eulerian?

Answer: Let's denote the complete bipartite graph as $K_{r,s}$. Therefore, the complete bipartite graph $K_{r,s}$ will be Eulerian if both r and s are even numbers.

∴ Complete bipartite graphs are Eulerian if the two disjoint set of vertices have even number of vertices.

e.g: $K_{2,2}$, $K_{6,6}$, $K_{12,10}$, ---

③

(iii) Which Platonic graphs are Eulerian?

Answer: Platonic graphs are Eulerian if each vertex has an even number of degree. Only the octahedron is Eulerian

This is because the Octahedron is the only platonic graph in which all the vertices have even degree

(iv) For which values of n is the wheel W_n Eulerian?

Answer: A wheel W_n has $2(n-1)$ edges for n vertices

Therefore the degree for each vertex in the cycle is 2, while the degree of the new vertex formed in the central part of the graph will be $n-1$. So the wheel will be eulerian if $n-1$ is even.

∴ W_n is Eulerian for values of n where $n-1$ is an even number. Eg: W_5, W_7, W_9, ---

(v) For which values of K is the K-cube Q_K Eulerian?

Answer: A cubic graph has a degree regular of K. Therefore, K-cube Q_K is Eulerian if K is an even number.

Eg: Q_4, Q_6, Q_8, ---

6.4) Let G be a connected graph with K (>0) vertices of odd degree.

(i) Show that the minimum number of trails, that together include every edge of G and that have no edges in common is K/2

Solution: G is a connected graph with K vertices of odd degree, then G must have an even number of vertices with odd ~~of~~ degrees, in order for G to meet the Eulerian trial condition for which no edges are in common and the trial includes all edges in G. We can assume this because the sum of all degrees in a graph is always even, based on the Hand shaking Lemma.

Let's assume there are 2m vertices with odd degree in G where m is a positive integer i.e m > 0. Since each trail will start and end at vertices with odd degrees, then there must be at least m trials for all ~~possible~~ vertices with odd degrees which is K (K = vertices with all degrees).

$\therefore 2m = K \quad , \quad m = K/2$

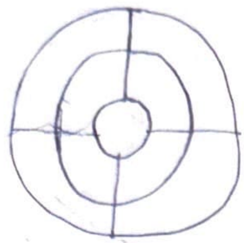
Therefore, the minimum number of trials is m which is $\frac{K}{2}$

∴ This shows that the minimum number of trials, that together include every edge of G and that have no edges in common is K/2

⑤

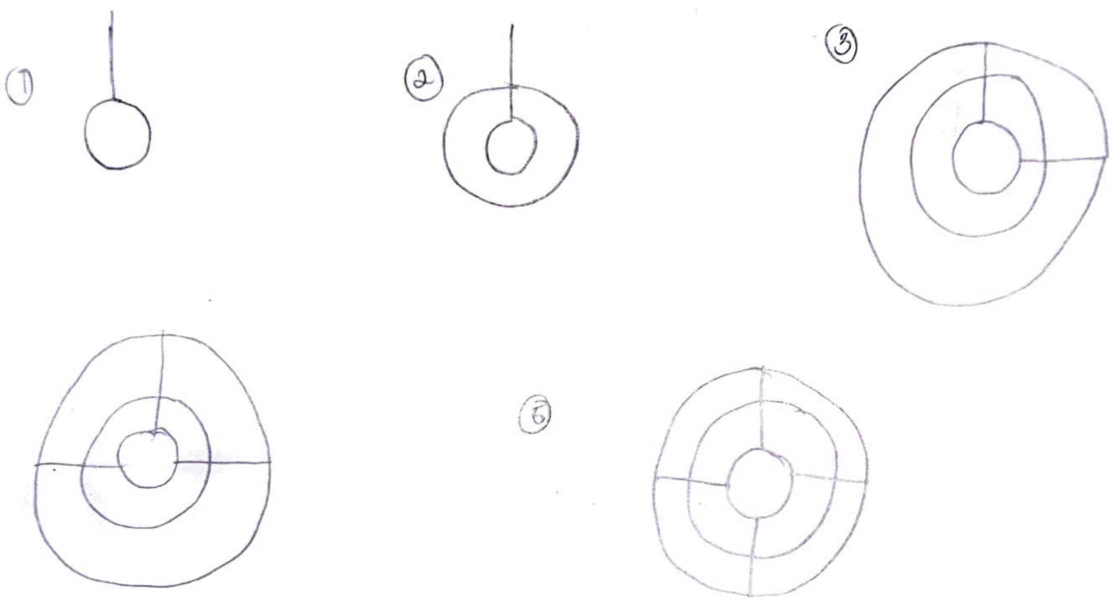
(ii) How many continuous pen-strokes are needed to draw the diagram in Fig 6.8 without repeating any line?

Fig 6.8 :



⇒ a continuous pen stroke involves creating a closed loop without lifting the pen from the paper.

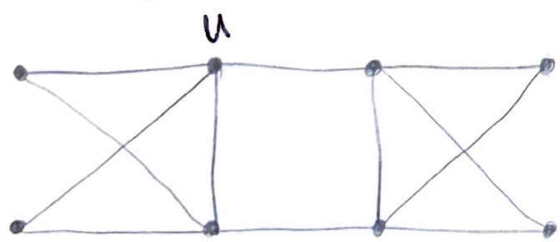
Solution:



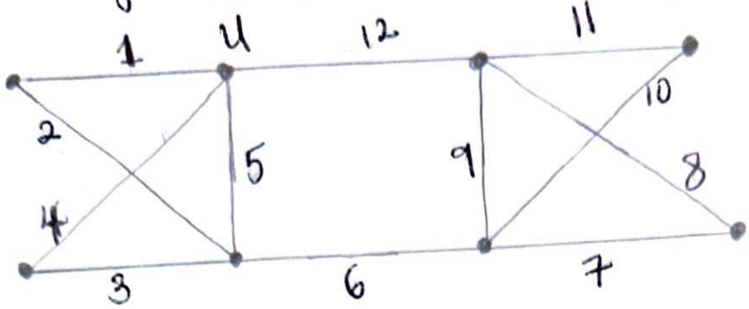
∴ There are 5 continuous pen-strokes needed to draw the diagram without repeating any line

6.5.) Use Fleury's algorithm to produce an Eulerian trail for the graph in Fig 6.9.

Fig 6.9:



Solution: start at any vertex u (this is already given in the diagram) then traverse the edges:



6.6) (i) Show that the line graph of a simple Eulerian graph is Eulerian.

Solution: A simple Eulerian graph ~~is~~ G is eulerian if it has an Eulerian Trail (i.e every edge is visited exactly once in the Close walk).

Therefore, to show that the line graph of a simple eulerian graph also has a closed walk that visits every edge exactly once, we can start a walk from an edge in G, and move to an adjacent edge on the line graph if the corresponding edges in G share a common vertex.

Since G is a eulerian graph and the line graph is of G, then we can construct a closed walk in the line graph, that covers every edge exactly once. This means that the line graph is also Eulerian. Therefore, the line graph of a simple Eulerian graph is Eulerian.

⑦

(ii) If the line graph of a simple graph G is Eulerian, must G be Eulerian?

Solution: Let's assume that the line graph of a simple graph G is Eulerian. This means we can construct a closed walk in the ~~thing~~ line graph that covers every edge exactly once. But there might be vertices along G that we cannot start and end at and will cover all the edges exactly once.

So, G may not be eulerian.

Therefore, if the line of graph of a simple graph G is Eulerian, it does not mean G is Eulerian.