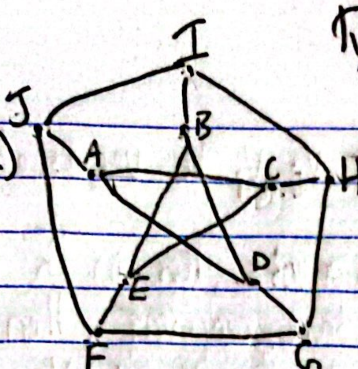
HW 5
Tyler Harper

5.1.)



i.) F-G-H-I-J-F

ii.) F-G-H-I-J-A-C-E-B-D

iii.) 5: F-G-H-I-J-F

6: I-B-D-A-C-H-I

9: I-B-D-A-C-E-F-G-H-I

8: I-B-D-G-F-E-C-H-I

iv.) {(J,A), (J,I), (J,F)}

{(J,I), (J,F), (A,D), (A,C)}

{(J,I), (J,F), (C,H), (C,E), (A,D)}

5.2.) i.) 3 ii.) 3 iii.) 7 iv.) 3 v.) 4 vi.) 5 vii.) 5

5.3.) Let V_0 be a vertex of component C_0. Let l_i be the distance of the shortest walk from V_0 to V_i. If l_i is even, color V_i black, otherwise color it white.
Repeat this process for every component of G.
If there were 2 adjacent white vertices or 2 adjacent black vertices, then there would be an odd numbered cycle. Therefore, there are no 2 adjacent vertices of the same color, therefore G is bipartite

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5.4.) Let G be disconnected. Let v_i, v_j be vertices of G.

If v_i, v_j are in different components, then in $\bar{G}$ they are adjacent. If they are in the same component, then let v_k be in the other component. Then in $\bar{G}$, $v_i - v_k - v_j$ is a path. Therefore $\bar{G}$ is connected

5.5.) i.) $\lambda(C_6)=2$ $K(C_6)=1$

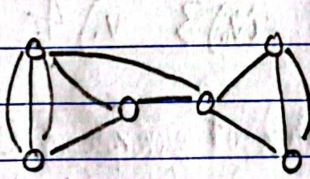
ii.) $\lambda(W_6)=3$ $K(W_6)=2$

iii.) $\lambda(K_{4,7})=4$ $K(K_{4,7})=4$

iv.) $\lambda(Q_4)=4$ $K(Q_4)=4$

5.6.) i.) If the minimum degree is k, then if you cut all k edges connected to that vertex, then that vertex is disconnected

ii.)



$K(G)=1$

$\lambda(G)=2$

$k=3$

$\Rightarrow$

5.7.) i.) Suppose there is 2 vertices u and v s.t. there are no cycles C with $u,v \in C$. Since G is connected, there is a path from u to v. Since u,v are not in a cycle, we cut one edge from the path to disconnect the graph. Then G is not 2-connected ※.

Therefore each pair of vertices are part of a common cycle.

$\Leftarrow$

Let $u,v \in C$. Cut a vertex adjacent to u. Since u,v are in a cycle, then they are still connected, so G is 2-connected □

ii.) A graph G is 2-edge-connected iff any 2 distinct vertices are joined by at least 2 paths with no edge in common.

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5.8.) i) The ij^{th} entry of A^2 is

$$\sum_{k=1}^{n} A_{ik} \cdot A_{kj}$$

each of these entries are either 1 or 0. If there exists a path from v_i to some vertex v_k to v_j, i.e. a walk of length 2, then A_{ik} and $A_{kj} = 1$, so it will be counted in the sum.

Therefore A^2_{ij} will be the count of all 2 length walks.

ii.) $A_{ij} = A_{ji}$, so $A^2_{ii} = \sum_{k=1}^{n} A_{ik}$. i.e. A^2_{ii} is the degree of the vertex v_i. The sum of the degrees of all vertices of a graph G is $2m$, so the sum of the diagonal of A^2 is $2m$.

iii.) A^3_{ij} is the number of 3 length walks from v_i to v_j.

A^3_{ii} is the number of walks of length 3 from v_i to v_i, which is a triangle. However a triangle can be walked from 2 different directions. Since there are 3 different vertices for each triangle, each triangle is counted $2 \cdot 3 = 6$ times, so the sum of the diagonals are $6t$.

5.9.) i.) $d(v,w) \geq 2$, so there exists a vertex z in the walk. Since $d(v,w)$ is the shortest path, $d(v,z)$ and $d(z,w)$ are are lengths of the 2 subpaths, so $d(v,z) + d(z,w) = d(v,w)$