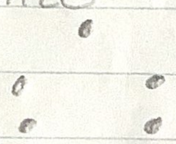
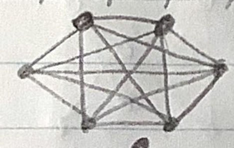
April Prinzo HW3 3.1, 3.2, 3.3, 3.4, 3.5, 3.6, 3.7, 3.8

3.1) i) N_5:

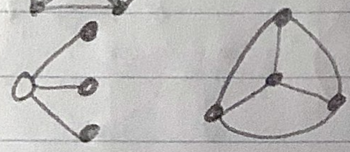


ii) K_6:

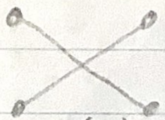


iii) $K_{2,4}$:

iv) $K_{1,3} \cup W_4$



C_4:  v) $\overline{C_4}$

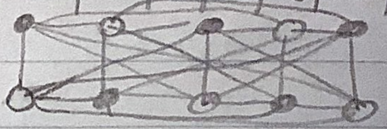


3.2) i) K_{10} has $\frac{10(9)}{2} = 45$ edges
ii) $K_{5,7}$ has $5 \times 7 = 35$ edges
iii) Q_4 has $4 \times 2^3 = 32$ edges
iv) W_8 has $7 + 7 = 14$ edges
v) the petersen graph has 15 edges

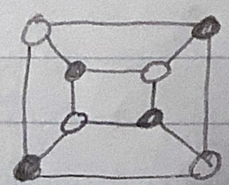
3.3) Tetrahedron: 4 vertices 6 edges
Octahedron: 6 vertices 12 edges
Cube: 8 vertices 12 edges
Icosahedron: 12 vertices 30 edges
Dodecahedron: 20 vertices 30 edges

3.4) Regular Graphs: 1, 2, 4, 8, 10, 18, 31
Bipartite Graphs: 2, 3, 5, 6, 8, 11, 12, 13, 17, 23

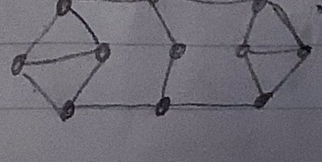
3.5) i) $K_{5,5}$



ii) Cube:

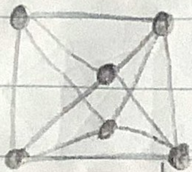


iii) The graph does not exist
iv) The graph does not exist



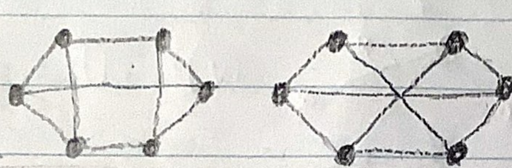
X Not Cubic

3.5) v)



3.6) 4

6



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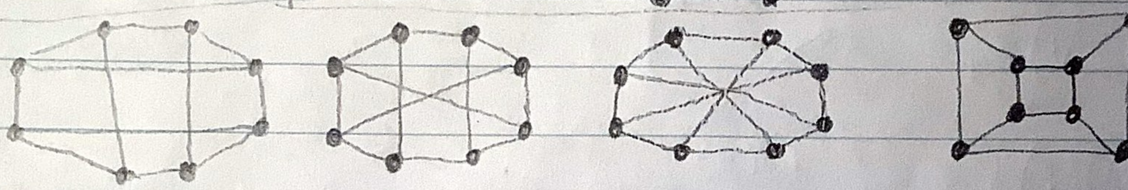
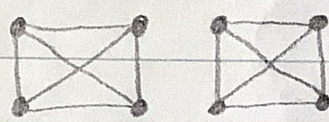
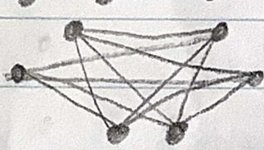
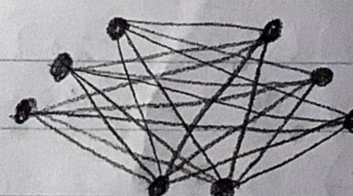




3.7) $K_{2,2,2}$



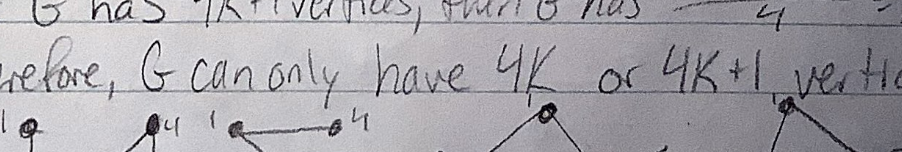
$K_{3,3,2}$



$K_{3,4,5}$ has $12 + 15 + 20 = 47$ edges

3.8) i) Proof. Let G be a self-complementary graph. Then the number of edges of G must equal half of the total possible number of edges, or $\frac{(n)(n-1)}{4}$. Let $K \in \mathbb{Z}$. If G has $4K+2$ vertices, then G has $\frac{(4K+2)(4K+1)}{4}$ edges, which is not possible. If G has $4K+3$ vertices then G has $\frac{(4K+3)(4K+2)}{4}$ edges, which is not possible. If G has $4K$ vertices, then G has $\frac{(4K)(4K-1)}{4} = K(4K-1)$ edges. If G has $4K+1$ vertices, then G has $\frac{(4K+1)(4K)}{4} = K(4K+1)$ edges. Therefore, G can only have $4K$ or $4K+1$ vertices □

ii)



iii)