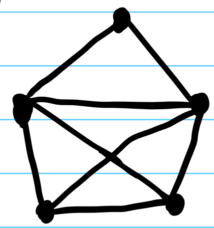
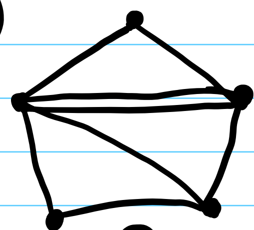
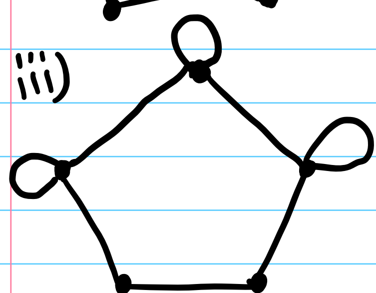
2.2) i)



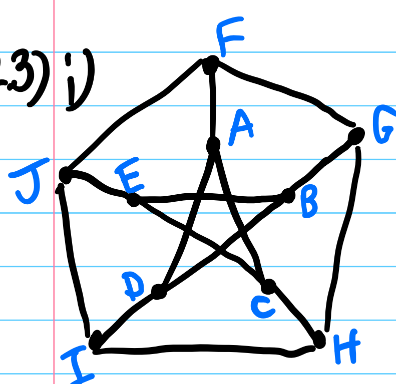
ii)

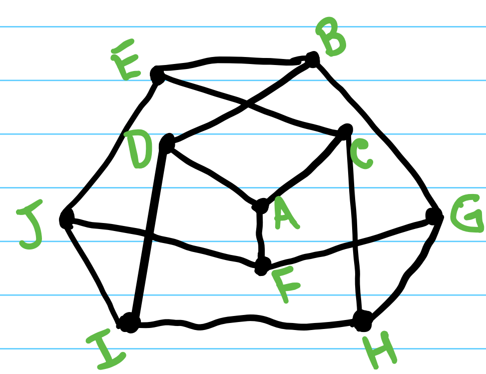


iii)

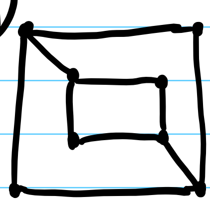


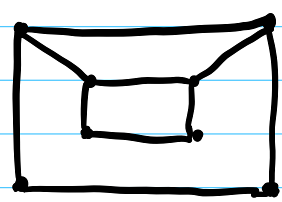
2.3) i)





ii)





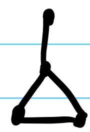
Not isomorphic because we cannot find a bijection between the edges: the graph on the left has edges going from the inner square to the outer square on opposite sides, the graph on the right has these edges on adjacent sides.

2.4) i) True: two graphs are isomorphic $\Rightarrow$ we have a bijection between their edges and vertices $\Rightarrow$ for some vertex v, $F(v)$ has same number of edges $\Rightarrow$ the two graphs have the same degree sequence

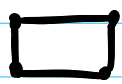
ii) False: see 2.3 part (ii) for two graphs with the same degree sequence which are not isomorphic

2.7) 

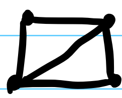
degree sequence $= (1,1,2,2)$; # of edges $=3$; $1+1+2+2 = 2\cdot 3$ ✓



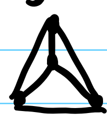
deg seq $= (1,2,2,3)$; # of edges $=4$; $1+2+2+3 = 2\cdot 4$ ✓



deg seq $= (2,2,2,2)$; # of edges $=4$, $2+2+2+2 = 2\cdot 4$ ✓



deg seq $= (2,2,3,3)$; # of edges $=5$; $2+2+3+3 = 2\cdot 5$ ✓



deg seq $= (3,3,3,3)$; # of edges $=6$; $3+3+3+3 = 2\cdot 6$ ✓

2.9) Suppose G is a simple graph with at least two vertices.

Case 1: No vertex has degree 0.

$\Rightarrow$ The degree sequence is $1 \le d_1 \le \dots \le d_n \le n-1$, if there are n vertices.

By the pigeonhole principle, at least two vertices have the same degree.

Case 2: one vertex has degree 0.

Consider the graph G' which is G without this one vertex. Then at least two vertices have the same degree in G' by case 1.

Case 3: At least two vertices have degree 0 (a.k.a. have same degre we are done)

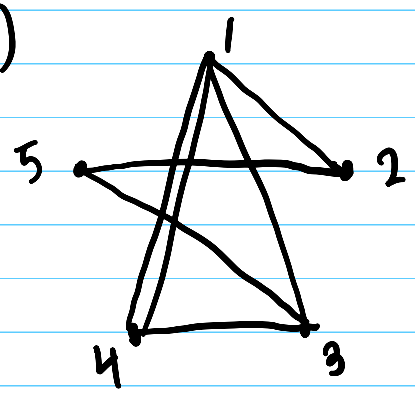
2.12) Adjacency matrix:

	1	2	3	4	5
1	0	1	0	0	1
2	1	0	1	0	1
3	0	1	0	2	0
4	0	0	2	0	1
5	1	1	0	1	0

Incidence matrix:

	1	2	3	4	5	6	7
1	1	1	0	0	0	0	0
2	0	1	1	0	1	0	0
3	0	0	0	0	1	1	1
4	0	0	0	1	0	1	1
5	1	0	1	1	0	0	0

2.13) i)



ii)