Generalizations of Conway's Subprime Fibonacci Sequences

Abrar Almahmeed, Pablo Blanco

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Introduction

- Conway's subprime Fibonacci sequences combine ideas from:
  - Fibonacci-type recurrences
  - Collatz-type behavior
- We investigate:
  - cycle formation
  - increasing/decreasing behavior
  - long-term patterns

Definition

The Conway's subprime Fibonacci sequences is defined by

$$a_n = \frac{c_1 a_{n-1} + c_2 a_{n-2}}{B(c_1 a_{n-1} + c_2 a_{n-2})}$$

where

$$B(x) = \begin{cases} 1 & \text{if } x \text{ is prime or } |x| \leq 1 \\ \operatorname{lpf}(x) & \text{otherwise} \end{cases}$$

- $a_0, a_1 c_1, c_2 \in \mathbb{Z}$
- $\operatorname{lpf}(x) =$ least prime factor of x

Goals of the Project

- Implement the sequences in Maple
- Experiment with different:
  - initial values a_0, a_1
  - parameters c_1, c_2
- Detect:
  - cycles
  - growth patterns

Maple Procedures

- `SubFseq(a0,a1,N,c1,c2)`: generates the subprime Fibonacci sequence corresponding to the initial values a_0, a_1 and parameters c_1, c_2, up to N terms.
- `SubpFCycles(M,N,c1,c2)`: computes the cycle information for all initial values in the range $-M \leq a_0, a_1 \leq M$.
- `FindCycle(S)`: detects cycles in a given sequence S.
- `CycleLengthSeq(M,N,c1,c2)`: computes the set of all possible cycle lengths for initial values in the range $-M \leq a_0, a_1 \leq M$.

Results and Observations: Case 1: $c_1 = c_2 = 1$

The observed cycle lengths are $\{1, 18, 19, 56, 136\}$

Main observations:

- When $(a_0, a_1) = (a, a)$:

  cycle length 1

- When $(a_0, a_1) = (a, -a)$:
  most sequences produce a cycle of length 1.
  Exceptions occur for

  $$a = \pm 1, \pm 8, \pm 16, \pm 24, \pm 32, \pm 48$$

  where the sequence instead forms a cycle of length 18.

Results and Observations: Case 2: $c_1 = c_2 = c$ with $c = 2, 3, 4$

- Positive initial values:

  increasing behavior

- Negative initial values:

  decreasing behavior

- No periodic behavior was detected within the tested range.

Results and Observations: Case 3: $c_1 = c_2 = c < 0$ with c up to -17

- For $c = -1, -2, -3$, the possible cycle lengths are $\{1, 3\}$.
- For prime negative values $c = -5, -7, \ldots, -17$, the possible cycle lengths are $\{1, 3, FAIL\}$
- For composite values $c = -4, -6, \ldots, -16$, no cycle is detected.
- The initial value $(0, 0)$ leads to a cycle of length 1.

Results and Observations: Case 4: $c_1 = -c_2$ with $c_1 = 1, 2, \ldots, 13$

- For $c_1 = 1, 2, 3$, the possible cycle lengths are $\{1, 6\}$.
- For prime values $c_1 = 5, 7, 11, 13$, the possible cycle lengths are $\{1, 6, \mathit{FAIL}\}$
- For composite values $c_1 = 4, 6, 8, 9, 10, 12$, no cycle is detected.
- The initial value $(0, 0)$ leads to a cycle of length 1.

Results and Observations: Case 5: $c_1 = -1$ and $c_2 = 1$

- The possible cycle lengths are $\{1, 2, 18, 56, 136\}$
- For initial values $(a_0, a_1) = (a, a)$, we observed that most sequences fall into cycles of length 2. In a smaller number,a cycle of length 18 appears with the following initial values {(-48, -48), (-32, -32),(-24, -24), (-16, -16), (-8, -8),(-1, -1), (1, 1), (8, 8), (16, 16), (24, 24), (32, 32), (48, 48)}
- For initial values of the form $(a_0, a_1) = (a, -a)$, the sequence enters a cycle of length 2.

Conclusion

- The behavior strongly depends on:
  - parameters
  - initial values
- Different choices produce:
  - cycles
  - increasing sequences
  - chaotic behavior
- Further work could include extending the computational range, improving efficiency for larger values, and investigating whether these observed patterns can be proven.