Statistical Properties of Permutations and Standard Young Tableau reflected by the Robinson–Schensted Algorithm

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1 Introduction

The Robinson–Schensted correspondence is a classical bijection between permutations $\pi \in S_n$ and pairs (P, Q) of standard Young tableaux of the same shape. It translates permutation statistics into tableau statistics. In particular, Schensted's theorem states that the length of the longest increasing subsequence of π is the first row length of the common shape, while the length of the longest decreasing subsequence is the first column length [2].

In this project, we study statistical properties of the Robinson–Schensted correspondence through Maple experiments. For each permutation π, we compute

$$\mathrm{RS}(\pi) = (P(\pi), Q(\pi)), \qquad \lambda(\pi) = \mathrm{shape}(P(\pi)) = \mathrm{shape}(Q(\pi)),$$

and investigate statistics of the shape $\lambda(\pi)$ and of the entries of $P(\pi)$ and $Q(\pi)$.

We first test the Logan–Shepp limit shape theorem for random Young diagrams under Plancherel measure [4]. Our computations compare scaled random Young diagrams with the limiting curve and verify numerically that the first row and first column lengths are asymptotic to $2\sqrt{n}$.

We then study the relationship between the first row and first column of the RS shape, equivalently the longest increasing and decreasing subsequence lengths of a random permutation. The data suggest negative finite-size covariance, while the correlation appears to approach 0 as n grows.

Next, we investigate the location of a fixed entry m in the insertion and recording tableaux. The marginal distribution of this location is known from McKay–Morse–Wilf [9]. We compute exact joint probabilities and observe evidence for an asymptotic independence phenomenon.

Finally, we introduce row and column sum vectors of standard Young tableaux. These numerical invariants lead to questions about when a tableau is uniquely determined by its row sums, column sums, or by the pair of both vectors.

2 Background on Robinson–Schensted Correspondence

The Robinson–Schensted correspondence is a bijection between permutations $\pi \in S_n$ and pairs (P, Q) of standard Young tableaux of the same shape. We write

$$\mathrm{RS}(\pi) = (P(\pi), Q(\pi)).$$

Here $P(\pi)$ is called the insertion tableau and $Q(\pi)$ is called the recording tableau. Both tableaux have the same shape, which we denote by

$$\lambda(\pi) = \mathrm{shape}(P(\pi)) = \mathrm{shape}(Q(\pi)).$$

Thus the Robinson–Schensted correspondence translates information about a permutation into information about the shape and entries of two standard Young tableaux.

A standard Young tableau is a filling of the boxes of a Young diagram with the numbers $1, 2, \ldots, n$, such that entries increase from left to right along each row and from top to bottom along each column. For example,

1	3	5
2	4	

is a standard Young tableau of shape $(3, 2)$. In this paper, we often represent the shape of a tableau as a vector of row lengths. Thus the shape of the above tableau is $(3, 2)$.

One of the most important features of the Robinson–Schensted correspondence is its relationship with increasing and decreasing subsequences of a permutation. The following classical theorem of Schensted gives this connection.

Theorem 2.1 (Schensted's theorem)**.** *Let $\pi \in S_n$, and let*

$$\mathrm{RS}(\pi) = (P(\pi), Q(\pi)).$$

If

$$\lambda = \mathrm{shape}(P(\pi)) = \mathrm{shape}(Q(\pi)),$$

then the length of the longest increasing subsequence of π is equal to the length of the first row of λ, and the length of the longest decreasing subsequence of π is equal to the length of the first column of λ. In other words,

$$\mathrm{LIS}(\pi) = \lambda_1, \qquad \mathrm{LDS}(\pi) = \lambda_1',$$

where λ_1 is the first row length and λ_1' is the first column length.

This theorem is the main reason that the Robinson–Schensted correspondence is useful for studying statistical properties of random permutations. Instead of studying the longest increasing or decreasing subsequences directly, one can study the first row and first column of the associated Young diagram.

In our computations, we use Maple to apply the Robinson–Schensted correspondence to permutations and record the resulting tableaux P and Q. The procedure `PropertyVerifier.txt` checks Schensted's theorem across all $72,873$ observations in our dataset by verifying that the longest increasing subsequence agrees with the first row length, and that the longest decreasing subsequence agrees with the first column length.

When π is chosen uniformly at random from S_n, the random shape $\lambda(\pi)$ is distributed according to Plancherel measure:

$$\mathbb{P}(\lambda) = \frac{(f^\lambda)^2}{n!},$$

where f^λ is the number of standard Young tableaux of shape λ. This probability distribution is the background for the Logan–Shepp limit shape theorem discussed in the next section.

3 Logan-Shepp Limit Shape

One of the classical results related to the Robinson–Schensted correspondence is the Logan–Shepp limit shape theorem for random Young diagrams. Let $\pi \in S_n$ be a uniformly random permutation, and let

$$RS(\pi) = (P(\pi), Q(\pi)).$$

The two tableaux $P(\pi)$ and $Q(\pi)$ have the same shape, which we denote by $\lambda(\pi)$. As π varies uniformly over S_n, the random partition $\lambda(\pi) \vdash n$ is distributed according to the Plancherel measure.

The Logan–Shepp theorem says that after scaling both axes by $\sqrt{n}$, the random Young diagram concentrates around a deterministic curve. In the coordinates used in our plots, this curve is given parametrically by

$$x(\theta) = \frac{2}{\pi}\left(\sin\theta - \theta\cos\theta\right),$$

and

$$y(\theta) = \frac{2}{\pi}\left(\sin\theta + (\pi - \theta)\cos\theta\right), \qquad 0 \le \theta \le \pi.$$

The curve starts at $(0, 2)$ and ends at $(2, 0)$. Thus the limit shape predicts that both the first row length and the first column length are asymptotically of order $2\sqrt{n}$:

$$\frac{\lambda_1}{\sqrt{n}} \longrightarrow 2, \qquad \frac{\lambda_1'}{\sqrt{n}} \longrightarrow 2.$$

Here λ_1 is the first row length and λ_1' is the first column length, equivalently the number of rows of λ.

To test this experimentally, we implemented Maple procedures that generate random permutations, compute their Robinson–Schensted shapes, scale the resulting Young diagrams by $\sqrt{n}$, and plot them together with the Logan–Shepp curve. The procedure `RandShape(n)` generates a random permutation in S_n and returns the shape of its insertion tableau. The procedure `ScaledShapePoints(L)` converts a partition L into the boundary points of its scaled Young diagram. Finally, `PlotLS(n)` displays the scaled random Young diagram together with the theoretical Logan–Shepp curve.

Figures 1 shows the result for $n = 100, 500, 2000$. The black staircase is the scaled Young diagram, and the red curve is the Logan–Shepp limit shape. For $n = 100$, the random diagram is still visibly noisy, but it already follows the general profile of the limit curve. For $n = 500$, the agreement is much clearer. For $n = 2000$, the staircase is very close to the limiting curve over most of the interval. This provides a visual confirmation of the convergence predicted by the Logan–Shepp theorem.

We also tested the convergence of the first row and first column numerically. For each n, we generated 1000 random permutations and computed the sample means of λ_1 and λ_1'. The procedure `FirstRowColMonte(n,K)` returns the sample means, variances, covariance, and correlation of the first row and first column over K trials.

n	K	$\mathbb{E}(\lambda_1)$	$\mathbb{E}(\lambda_1')$	$\mathbb{E}(\lambda_1)/\sqrt{n}$	$\mathbb{E}(\lambda_1')/\sqrt{n}$
100	1000	16.744	16.764	1.6744	1.6764
500	1000	40.307	40.197	1.8026	1.7977
1000	1000	58.278	58.247	1.8429	1.8419

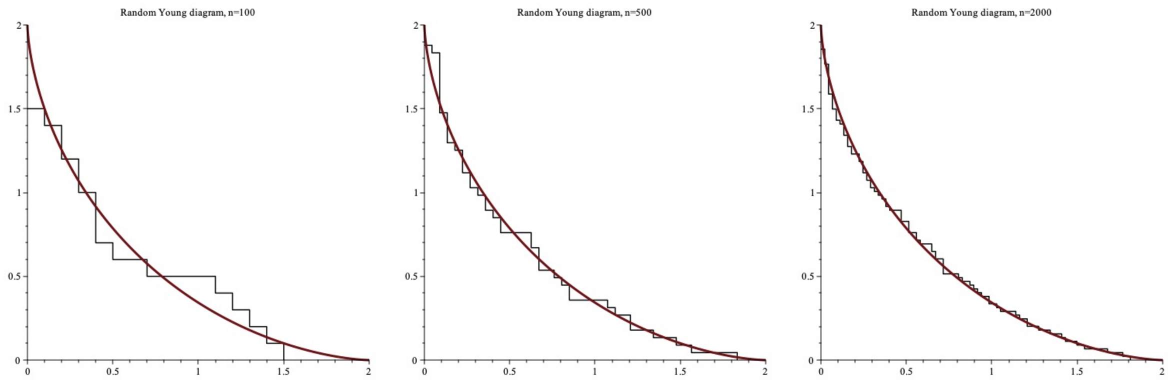


Figure 1: Scaled random Young diagrams compared with the Logan–Shepp limit shape for $n = 100, 500, 2000$.

The last two columns should approach 2 as n becomes large. The data show this trend: the normalized first row and first column averages increase from about 1.67 when $n = 100$ to about 1.84 when $n = 1000$. The convergence is not expected to be extremely fast at the edge of the diagram, but the numerical trend is consistent with the asymptotic prediction

$$\lambda_1 \sim 2\sqrt{n}, \qquad \lambda_1' \sim 2\sqrt{n}.$$

4 Covariance and correlation between the first row and column

One of the central reasons that the Robinson–Schensted correspondence is important in the study of random permutations is Schensted's theorem. Given a permutation $\pi \in S_n$, let

$$RS(\pi) = (P(\pi), Q(\pi)),$$

and let $\lambda = \text{shape}(P(\pi)) = \text{shape}(Q(\pi))$. Schensted's theorem states that the length of the longest increasing subsequence of π is equal to the length of the first row of λ, while the length of the longest decreasing subsequence is equal to the length of the first column of λ. In other words, if

$$R_n = \lambda_1, \qquad C_n = \lambda_1' = \ell(\lambda),$$

then

$$R_n = \text{LIS}(\pi), \qquad C_n = \text{LDS}(\pi).$$

The asymptotic behavior of these quantities is closely related to the Logan–Shepp limit shape. In their paper, Logan and Shepp studied the typical shape of a Young diagram obtained from a uniformly random permutation through the Robinson–Schensted correspondence. Their result implies that, after scaling by $\sqrt{n}$, the first row and first column both approach the value 2. That is,

$$\frac{R_n}{\sqrt{n}} \to 2, \qquad \frac{C_n}{\sqrt{n}} \to 2.$$

In the previous section, we verified this behavior experimentally using random permutations and the Robinson–Schensted algorithm.

A natural next question is to ask how the first row and first column are related to each other. Since the Young diagram has fixed area n, one might expect a competition between horizontal and vertical growth. If the first row is unusually long, then the diagram is more stretched in the horizontal direction; this should make it less likely for the first column to also be unusually long. Therefore, we expect the covariance between R_n and C_n to be negative.

We investigate this question experimentally. Recall that

$$\mathrm{Cov}(R_n, C_n) = \mathbb{E}[R_n C_n] - \mathbb{E}[R_n]\mathbb{E}[C_n],$$

and

$$\mathrm{Corr}(R_n, C_n) = \frac{\mathrm{Cov}(R_n, C_n)}{\sqrt{\mathrm{Var}(R_n)\,\mathrm{Var}(C_n)}}.$$

Using our Maple procedure `FirstRowColMonte(n,K)`, we generated K random permutations in S_n, computed their Robinson–Schensted shapes, and recorded the first row length R_n and first column length C_n. The procedure then estimates the expectations, variances, covariance, and correlation.

The following table shows the output for $K = 1000$ samples.

n	K	$\mathbb{E}[R_n]$	$\mathbb{E}[C_n]$	$\mathrm{Var}(R_n)$	$\mathrm{Var}(C_n)$	$\mathrm{Cov}(R_n, C_n)$	$\mathrm{Corr}(R_n, C_n)$
100	1000	16.7180	16.6190	2.5805	2.6778	−0.6364	−0.2421
500	1000	40.1910	40.2380	5.2705	5.6394	−0.6685	−0.1226
1000	1000	58.2380	58.1340	7.0354	7.8600	−0.8889	−0.1195

In all three cases, the estimated covariance is negative. This supports the intuition that the first row and first column are negatively related for finite n. However, the estimated correlation becomes closer to zero as n increases. This suggests that although the first row and first column have a negative finite-size dependence, this dependence may become weaker asymptotically.

Based on these computations, we formulate the following conjectures.

Conjecture 4.1. *For every $n \geq 2$,*

$$\mathrm{Cov}(R_n, C_n) < 0.$$

Equivalently, for a uniformly random permutation $\pi \in S_n$, the lengths of the longest increasing and longest decreasing subsequences are negatively correlated.

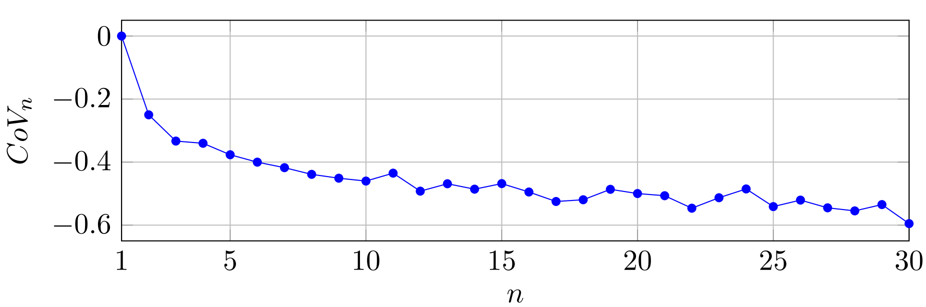
Conjecture 4.2. *As $n \to \infty$,*

$$\mathrm{Corr}(R_n, C_n) \to 0.$$

Conjecture 4.3. *$\exists k$ s.t,*

$$\lim_{n\to\infty} \mathrm{Cov}(R_n, C_n) = k.$$

Covariance of Monotone Subsequence Lengths by Permutation Size



Thus our data suggests two phenomena: first, the first row and first column appear to be negatively correlated for every finite n; second, the strength of this correlation appears to decrease as n grows. This is consistent with the Logan–Shepp limit shape, where both the first row and first column are asymptotically of size $2\sqrt{n}$, while their fluctuations become relatively small compared with the leading-order shape.

5 A Experimental Row-Removal Heuristic

5.1 Procedures:

Maple package AvgShape.txt contains procedures to calculate the average shape of a standard Young tableaux outputted by the Robinson–Schensted correspondence. SYTShape() finds the shape of a standard Young tableaux and outputs it in vector representation. We then pick with replacement to generate 3000 random observations of standard Young tableaux shapes for each n from 1 to 30. AvgnthLength(x) and senthLength(x) find the average and standard error respectively of row $1 \leq x \leq 30$ for each n from 1 to 30.

5.2 Expanded Logic for Row 2:

Cut-Paste: Cutting-out any row from a standard Young tableaux and pasting together all rows before and after in the same order always results in a valid standard Young tableaux after re-numerating by size of each element.

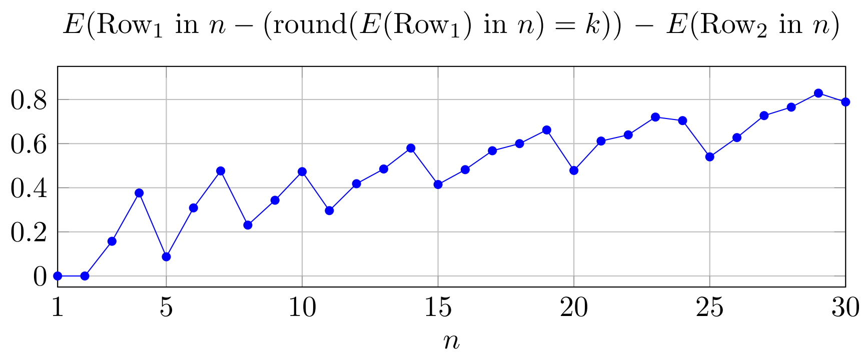
A somewhat natural sounding assumption follows that the length of row 2 in P is the length of the longest increasing subsequence in π that doesn't use any elements contained in the true longest increasing subsequence, however this is incorrect.

Proposition 5.1. *Denote the amount of elements deleted from the standard Young tableaux by Cut-Paste to be k, then the resulting standard Young tableaux from the Cut-Paste algorithm starting with input P, of size n from the Robinson–Schensted correspondence does not have the same distribution as P of size $n-k$ generated from the correspondence.*

Heuristic explanation. This result is easy to verify by imaging a recursive Cut-Paste process on all row_i $i > 1$, the end result is a standard Young tableaux that only occupies 1 row with probability 100% and has arbitrarily many entries, however no standard Young tableaux generated by the Robinson–Schensted correspondence with $n \geq 2$ shares this distribution. □

5.3 Evidence of Proposition:

TestProposition() rounds the size of the average first row in a standard Young tableaux generated by the Robinson–Schensted correspondence of size n to the closest integer k. It then compares the size of the average first row in a standard Young tableaux generated by the Robinson–Schensted correspondence of size $n - k$ of one of size n. The result appears to be an upwards and increasing in n bias towards the size of the average first row in $n - k$.



Footnote: It's worthwhile to observe that $n = 30$ was not sufficiently large enough to bring this difference above 1, if it surpasses that value or not may be of interest.

6 Location statistics of a distinguished entry under RS

Let $\pi \in S_n$ be a uniformly random permutation, and let

$$RS(\pi) = (P(\pi), Q(\pi))$$

be its Robinson–Schensted image. Here $P(\pi)$ is the insertion tableau and $Q(\pi)$ is the recording tableau. For $1 \leq m \leq n$, define

$$X^P_{n,m} = \text{the cell containing the value } m \text{ in } P(\pi),$$

and

$$X^Q_{n,m} = \text{the cell containing the label } m \text{ in } Q(\pi).$$

Although $X^P_{n,m}$ and $X^Q_{n,m}$ are generally different random variables, their marginal distributions are the same.

We first consider the marginal probability

$$p_m(i,j) = \mathbb{P}\left(X^P_{n,m} = (i,j)\right) = \mathbb{P}\left(X^Q_{n,m} = (i,j)\right).$$

The probability is independent of n once $n \geq m$. Indeed, the location of the label m in $Q(\pi)$ is determined by the first m insertions, while the location of the value m in $P(\pi)$ is determined by the relative order of the values $1, \ldots, m$ in π. In both cases, for a uniformly random $\pi \in S_n$, the relevant relative order is uniformly distributed over S_m. Therefore it is enough to study the case $n = m$.

Let $\lambda \vdash m$ be a partition of m, and let f^λ denote the number of standard Young tableaux of shape λ. For a cell $c = (i,j)$, let $\mathrm{Cor}(\lambda)$ denote the set of removable corners of λ. Then the marginal distribution is given by

$$p_m(i,j) = \frac{1}{m!} \sum_{\substack{\lambda \vdash m \\ (i,j) \in \mathrm{Cor}(\lambda)}} f^\lambda f^{\lambda \setminus (i,j)}.$$

This formula is a special case of the entry-distribution formulas for standard Young tableaux studied by McKay–Morse–Wilf [9].

Proposition 6.1. *For $n \geq m$, the marginal distribution of the location of m is given by*

$$p_m(i,j) = \frac{1}{m!} \sum_{\substack{\lambda \vdash m \\ (i,j) \in \mathrm{Cor}(\lambda)}} f^\lambda f^{\lambda \setminus (i,j)}.$$

Proof. It suffices to compute the probability that the entry m lies in cell $c = (i,j)$ in the recording tableau $Q(\sigma)$ for a uniformly random permutation $\sigma \in S_m$.

By the Robinson–Schensted correspondence, permutations in S_m are in bijection with pairs (P,Q) of standard Young tableaux of the same shape $\lambda \vdash m$. For a fixed shape λ, the insertion tableau P can be chosen in f^λ ways.

Now consider the recording tableau Q. Since m is the largest entry, it can only appear in a removable corner of λ. If $c \notin \mathrm{Cor}(\lambda)$, then there are no such tableaux. If $c \in \mathrm{Cor}(\lambda)$, then deleting the cell c containing m gives an arbitrary standard Young tableau of shape $\lambda \setminus c$. Thus the number of possible recording tableaux with m in cell c is $f^{\lambda \setminus c}$.

Therefore, for this fixed shape λ, the number of RS pairs (P,Q) with m in cell c of Q is

$$f^\lambda f^{\lambda \setminus c}.$$

Summing over all $\lambda \vdash m$ and dividing by $m!$ gives the desired formula. □

The marginal distribution $p_m(i,j)$ is therefore quite well understood. Moreover, since it is independent of n, it does not capture how the two tableaux $P(\pi)$ and $Q(\pi)$ interact inside the Robinson–Schensted correspondence. To study this dependence, we next consider the joint law of the two locations

$$\left(X^P_{n,m}, X^Q_{n,m}\right).$$

For cells c and d, define

$$J_{n,m}(c,d) = \mathbb{P}\left(X^P_{n,m} = c,\ X^Q_{n,m} = d\right).$$

Unlike the marginal probability $p_m(c)$, the joint probability $J_{n,m}(c,d)$ generally depends on n.

For a partition $\lambda \vdash n$, define

$$A_{\lambda,m}(c) = \#\{T \in \mathrm{SYT}(\lambda) : T(c) = m\}.$$

Thus $A_{\lambda,m}(c)$ counts the number of standard Young tableaux of shape λ in which the entry m appears in cell c.

Proposition 6.2. *For $1 \leq m \leq n$, the joint law is given by*

$$J_{n,m}(c,d) = \frac{1}{n!} \sum_{\lambda \vdash n} A_{\lambda,m}(c) A_{\lambda,m}(d).$$

Proof. By the Robinson–Schensted correspondence, permutations in S_n are in bijection with pairs (P, Q) of standard Young tableaux of the same shape $\lambda \vdash n$. Fix such a shape λ.

The number of possible insertion tableaux P of shape λ with the entry m in cell c is $A_{\lambda,m}(c)$. Similarly, the number of possible recording tableaux Q of shape λ with the entry m in cell d is $A_{\lambda,m}(d)$. Hence, for this fixed shape λ, the number of RS pairs satisfying both conditions is

$$A_{\lambda,m}(c) A_{\lambda,m}(d).$$

Summing over all partitions $\lambda \vdash n$ and dividing by the total number $n!$ of permutations in S_n gives the formula. ☐

Equivalently, the quantity $A_{\lambda,m}(c)$ can be expanded using skew tableaux. If $\mu \vdash m$ and $\mu \subseteq \lambda$, then the entries $1, \ldots, m$ occupy a subshape μ of λ. The condition that m lies in c means that c must be a removable corner of μ. Thus

$$A_{\lambda,m}(c) = \sum_{\substack{\mu \vdash m \\ \mu \subseteq \lambda \\ c \in \mathrm{Cor}(\mu)}} f^{\mu \setminus c} f^{\lambda/\mu},$$

where $f^{\lambda/\mu}$ denotes the number of standard Young tableaux of skew shape λ/μ.

The joint law therefore has an exact finite formula, but this formula depends on n. This leads to a natural asymptotic question: for fixed m and fixed cells c, d, do the two locations become independent as $n \to \infty$? In other words, do we have

$$J_{n,m}(c,d) \longrightarrow p_m(c) p_m(d)?$$

Our computations suggest that the answer is yes, and that the convergence is at least of order n^{-2}.

Conjecture 6.3 (Asymptotic independence)**.** *Fix m and cells c, d. Then*

$$J_{n,m}(c,d) - p_m(c) p_m(d) = O_m\left(\frac{1}{n^2}\right) \qquad \text{as } n \to \infty.$$

Equivalently, for fixed m, c, d, the sequence

$$n^2 \left(J_{n,m}(c,d) - p_m(c) p_m(d)\right)$$

is bounded as $n \to \infty$.

We verified these conjectures using Maple. The procedure computes the exact marginal probabilities $p_m(c)$ and $p_m(d)$, then computes the joint probability $J_{n,m}(c,d)$ for various values of n. For each choice of m, c, d, we examine the scaled difference

$$n^2 \left(J_{n,m}(c,d) - p_m(c) p_m(d)\right).$$

The conjecture predicts that this quantity should remain bounded as n grows.

These computations indicate that although the marginal distribution of the location of a distinguished entry is already well understood, the joint dependence between its locations in the insertion and recording tableaux contains additional structure. The asymptotic independence phenomenon and the rate of convergence appear to provide a natural next direction for studying statistical properties of the Robinson–Schensted correspondence.

7 Row and Column Sums

7.1 Preliminaries:

Define a row sum of a standard Young tableaux as a vector with *i-th* entry equal to Σ Entries of Row_i. Similarly column sum as Σ Entries of Column_i.

The size of the input standard Young tableaux, n, into a row or column sum can simply be determined by summing the entries in the vector and setting that sum equal to $\frac{n(n+1)}{2}$. Using the known size of n, we can look at the set of all partitions of n to construct a set of candidate shapes of the initial standard Young tableaux.

7.2 Questions on Uniqueness of Sum Vectors:

Going back to the row or column sum and pairing it with the set of candidate shapes, we can show that for <u>some</u> vectors, $\exists!$ standard Young tableaux that generates it under the row or column sum algorithm. I.e. any row or column sum vector that reads lexicographically corresponds to the unique standard Young tableaux that is just a single row

1	2	3

or column

1
2
3

respectively.

However,

1	3	4
2	5	

and

1	2	5
3	4	

have the same row sum vector

8
7

.

It can be easily verified by brute force that this is the only such example of non-unique row sum vectors for standard Young tableaux with $n \leq 5$. Some natural questions then follow:

Question 7.1. *What proportion of standard Young tableaux have unique sum vectors?*

Question 7.2. *Which properties of a standard Young tableau determine a bijection with a sum vector?*

Question 7.3. *Does every standard Young tableau have a unique (row sum vector, column sum vector) pair?*

Question 3 can quickly be rejected,

1	2	6
3	4	7
5	8	

1	3	5
2	4	8
6	7	

the smallest counterexample.

Question 7.4. *For a fixed n, what is the maximum number of standard Young tableau described by the same sum vector?*

Question 7.5. *For a fixed n, what is the maximum number of standard Young tableau described by the same (row sum vector, column sum vector) pair?*

Consider the group:

1	2	4	10
3	5	9	11
6	7	12	
8	13		

1	2	4	10
3	6	8	11
5	7	13	
9	12		

1	2	5	9
3	6	7	12
4	8	13	
10	11		

1	2	6	8
3	5	7	13
4	9	12	
10	11		

1	3	4	9
2	6	8	12
5	7	13	
10	11		

Question 7.6. *What proportion of standard Young tableaux have unique (row sum vector, column sum vector) pairs?*

Question 7.7. *Which properties of a standard Young tableau determine a bijection with a (row sum vector, column sum vector) pair?*

Question 7.8. *Do all groups of standard Young tableau described by the same sum vector have the same shape?*

Question 7.9. *Do all groups of standard Young tableau described by the same (row sum vector, column sum vector) pair have the same shape?*

Questions 7.2, 7.7, 7.8, 7.9 are left unanswered for the remainder of this paper.

7.3 Procedures:

Maple package RSStatFinder.txt contains procedures to calculate statistics and properties of row sum and column sum vectors of P's & Q's from the the Robinson–Schensted correspondence.

MaxGroupBoth(n) calculates by brute-force the maximum number of standard Young tableau of size n described by the same row sum vector OR column sum vector and returns the numbers as a pair. It is easy to show these numbers are always equal by swapping the rows and columns of the standard Young tableau.

MaxGroupPair(n) calculates by brute-force the maximum number of standard Young tableau of size n described by the same (row sum vector, column sum vector) pair.

PropUniqueSYTRow(n) and PropUniqueSYTCol(n) calculates by brute-force the proportion of standard Young tableaux that have unique sum vectors. By the same swapping argument these should again be equal.

PropUniqueSYTPair(n) calculates by brute-force the proportion of standard Young tableaux that have unique (row sum vector, column sum vector) pairs.

7.4 Largest Sum-Vector Fiber over standard Young tableaux of size n:

We provide by exhaustive methods the partial sequences of answers for n=i for i from 1 to 13 to Questions 7.4 and 7.5.

MaxGroupBoth = 1, 1, 1, 1, 2, 2, 4, 6, 8, 14, 22, 49, 97
MaxGroupPair = 1, 1, 1, 1, 1, 1, 1, 2, 2, 2, 3, 3, 5

A search in OEIS yields no results for either sequence, making these likely novel integer sequences and the topic of Sum-Vector Fibers over standard Young tableaux of size n likely new and open.

7.5 Proportion of SYT with unique sum vectors:

We provide by exhaustive methods the partial sequences of answers for n=i for i from 1 to 11 to Questions 7.1 and 7.6.

n	Proportion unique by row sum	Proportion unique by column sum
1	1.000000	1.000000
2	1.000000	1.000000
3	1.000000	1.000000
4	1.000000	1.000000
5	0.923077	0.923077
6	0.815789	0.815789
7	0.698276	0.698276
8	0.566754	0.566754
9	0.449618	0.449618
10	0.347094	0.347094
11	0.260029	0.260029

Table 1: Proportion of standard Young tableaux of size n uniquely identified by their row sum vector and column sum vector respectively.

n	Proportion of SYTs with unique (R, C) pair
1	1.000000
2	1.000000
3	1.000000
4	1.000000
5	1.000000
6	1.000000
7	1.000000
8	0.994764
9	0.990840
10	0.984836
11	0.977252

Table 2: Proportion of standard Young tableaux of size n uniquely identified by their (row sum, column sum) pair.

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