Assignment 8

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2. Prove that (Defect,Defect) is always the unique Nash Equilbria, (including mixed one) of the version of Prisoner's dillema (called the "donation game" in Sigmund's book) Show that if T=R+0.01, R=10000P, S=P-0.01, In (1.1)) how tragic it is.

Problem 2:

(i)

Player 2

Player 1		
	R, R	S, T*
	T*, S	P*, P*

R = Reward
T = Temptation
C = Cooperate
D = defect

We have $T > R > P > S$.

If Player 1 chooses row 1 then Player 2 chooses T since $T > R$ so we mark T in (1, 2)

If player 1 chooses row 2 then Player 2 chooses P since $P > S$ so we mark P in (2, 2)

If player 2 chooses column 1 then player 1 chooses T since $T > R$. so we mark T in (2, 1)

If player 2 chooses column 2 then player 1 chooses P since $P > S$. so we mark P in (2, 2).

Hence, Nash equilibra is (P, P) which is (Defect, Defect).

(iii) $T = R + 0.01$, $R = 1000P$, $S = P - 0.01$

We have $T > R > P > S$.

$$1000P + 0.01 > 1000P > P > P - 0.01$$

Since the Nash Equilibrium is (Defect, Defect) = (P, P)

So, it will be 1000 times worse than choosing (R, R) = (Reward, Reward).

3. Prove that in the version of the Snwodrift game in section 1.4, with T>R>S>P, (Refuse,Pay) and (Pay,Refuse) are two pure Nash equilibria. Either by hand, or using Maple, prove that in addition there is a mixed Nash equilibrium. Find an explicit expression for that additional Nash equilibrium, in terms of the parameters T,R,S,P, and check it for random numerical values using MNE(G).

Problem 3:

(i)

	C	D
C	R , R	S^*, T^*
D	T^*, S^*	P , P

$$T > R > S > P$$

We use the same process as before and we notice that we have two Nash Equilibrium. They are

$(S, T) = (C, D) = (\text{Pay}, \text{Refuse})$

$(T, S) = (D, C) = (\text{Refuse}, \text{Pay})$.

(ii), (iii)

Player 2

Player 1		C p	D $(1-p)$	Payoff
C	q	R , R	S , T	$Rp + S(1-p)$
D	$(1-q)$	T , S	P , P	$Tp + P(1-p)$
Payoff		$Rq + S(1-q)$	$Tq + P(1-q)$	

Suppose player 1 has probability p and player 2 has probability q of winning.

So, $$u_1(C) = u_1(D)$$
$$Rp + S(1-p) = Tp + P(1-p)$$
$$Rp + S - Sp = Tp + P - Pp$$
$$Rp - Sp - Tp + Pp = P - S$$
$$p(R - S - T + P) = P - S$$
$$p = \frac{P-S}{R-S-T+P}$$

and $$u_2(C) = u_2(D)$$
$$Rq + S(1-q) = Tq + P(1-q)$$
$$Rq + S - Sq = Tq + P - Pq$$
$$Rq - Sq - Tq + Pq = P - S$$
$$q(R - S - T + P) = P - S$$
$$q = \frac{P-S}{R-S-T+P}$$

So there is a mixed Nash Equilibrium:

$$\left(\frac{P-S}{R-S-T+P}, \frac{P-S}{R-S-T+P} \right).$$

(iv) $$\begin{bmatrix} [4,4] & [3,5] \\ [5,3] & [1,1] \end{bmatrix} = \begin{bmatrix} [R,R] & [S,T] \\ [T,S] & [P,P] \end{bmatrix}$$

So, $T > R > S > P$ where

$T=5$, $R=4$, $S=3$, $P=1$. This means our mixed nash equilibrium is

$$\left(\frac{P-S}{R-S-T+P}, \frac{P-S}{R-S-T+P}\right) = \left(\frac{1-3}{4-3-5+1}, \frac{1-3}{4-3-5+1}\right)$$

$$= \left(\frac{-2}{-3}, \frac{-2}{-3}\right)$$

$$= \left(\frac{2}{3}, \frac{2}{3}\right)$$

which matches with MNE22(G) in maple.