The average number of i-dimensional implicants

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Conjecture 0.1. *ImplicantsAve(n,a,i)*=$\begin{pmatrix} a \\ 2^i \end{pmatrix} \frac{2^{n-i}\begin{pmatrix} n \\ i \end{pmatrix}}{\begin{pmatrix} 2^n \\ 2^i \end{pmatrix}}$

Proof. For each implicant of dimension $i > 0$, it came from 2 implicants of dimension i-1 such that these two implicants can be expressed as two sets of identical elements except one and the one with different signs. For example:

$$v_1 = \{x_1, \ldots, x_k, \ldots, x_{n-i+1}\}$$

$$v_2 = \{y_1, \ldots, y_k, \ldots, y_{n-i+1}\}$$

where $x_i = y_i, i = 1, \ldots, n-i+1, i \neq k$ and $x_k = -y_k$.

Therefore, an implicant of dimension i came from 2^i implicants of dimension 0 such that these 2^i implicants (expressed as sets) contain n-i identical elements and i elements with different combinations of signs. Let S be a random set of 0-dimensional implicants with n variables and $|S|$=a, we get that

$$ImplicantsAve(n, a, i) =$$

$$\sum_{s_1,\ldots,s_{2^i} \in S} Prob(s_1, \ldots, s_{2^i} \text{can be combined to one implicant})$$

Since $s_1, \ldots, s_{2^i}$ share n-i identical elements and i elements with different combination of signs, the probability in the above formula is $\frac{2^{n-i}\begin{pmatrix} n \\ i \end{pmatrix}}{\begin{pmatrix} 2^n \\ 2^i \end{pmatrix}}$ where $\begin{pmatrix} 2^n \\ 2^i \end{pmatrix}$ is the total number of ways to choose 2^i different sets, $\begin{pmatrix} n \\ i \end{pmatrix}$ is the number of

ways to choose the i elements with different combinations of signs and 2^{n-i} is the number of the combinations of n-i identical elements. Finally, we get that

$$ImplicantsAve(n, a, i) = \begin{pmatrix} a \\ 2^i \end{pmatrix} \frac{2^{n-i} \begin{pmatrix} n \\ i \end{pmatrix}}{\begin{pmatrix} 2^n \\ 2^i \end{pmatrix}}$$

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