3. We begin by searching/hoping for a nice numerical solution. identify(evalf()) on the numerator and denominator yields

$$4\,\pi^{(3/2)}$$

and

$$2\,\pi^{(3/2)}$$

respectively. This gives an answer of 2. identify() fails to find similar values when one of the factors is removed or changed, so we should expect that whatever method we shall use to prove it will *not* be extensible to the general case.

First, we utilize the Legendre relation by running simplify(mul(GAMMA(1/14+i/7), i=0..6), GAMMA, trig) to get

$$8\,\pi^{(7/2)}.$$

Knowing that

$$\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi},$$

we factor it out and find

$$\Gamma\left(\frac{1}{14}\right)\Gamma\left(\frac{9}{14}\right)\Gamma\left(\frac{11}{14}\right)\Gamma\left(\frac{3}{14}\right)\Gamma\left(\frac{5}{14}\right)\Gamma\left(\frac{13}{14}\right)=8\,\pi^3.$$

Now it suffices to find either the numerator or the denominator, which we do by utilizing the following special case of the Legendre relation:

$$\Gamma(x)=\frac{2^{(1/2-2x)}\sqrt{2}\sqrt{\pi}\,\Gamma(2x)}{\Gamma\left(x+\frac{1}{2}\right)}.$$

Strangely, evalb() returns *false* for this identity, and we conjecture that this is why Maple fails to simplify the expression as given. Converting the numerator, we obtain:

$$\Gamma\left(\frac{1}{14}\right)=\frac{2^{(6/7)}\sqrt{\pi}\,\Gamma\left(\frac{1}{7}\right)}{\Gamma\left(\frac{4}{7}\right)},$$

$$\Gamma\left(\frac{9}{14}\right)=\frac{2^{(5/7)}\sqrt{\pi}\,\Gamma\left(\frac{2}{7}\right)}{\Gamma\left(\frac{1}{7}\right)},$$

$$\Gamma\left(\frac{11}{14}\right)=\frac{2^{(3/7)}\sqrt{\pi}\,\Gamma\left(\frac{4}{7}\right)}{\Gamma\left(\frac{2}{7}\right)}.$$

Multiplying the right-hand sides easily yields

$$4\,\pi^{(3/2)},$$

which, when divided from our known product gives the expected value for the denominator

$$2\,\pi^{(3/2)},$$

and therefore,

$$\frac{\Gamma\left(\frac{1}{14}\right)\Gamma\left(\frac{9}{14}\right)\Gamma\left(\frac{11}{14}\right)}{\Gamma\left(\frac{3}{14}\right)\Gamma\left(\frac{5}{14}\right)\Gamma\left(\frac{13}{14}\right)}=2.$$