A reviewer report on "A symmetric chain decomposion of $L(5, n)$" by X. Wen.

Stanley [2] conjectured that Young's lattice $L(m, n)$ has a symmetric chain decomposition (SCD). Despite many efforts, this has been confirmed so far only for $\min(m, n) \leq 4$. As another step, this short manuscript presents an explicit construction of an SCD of $L(5, n)$. However, it does not bring any new combinatorial insight as it merely lists the chains in several tables and then leaves the task of verifying the decomposition to a computer program (although with a neat trick showing that these chains contributions to total weight lead to a correct generating function). The manuscript also omits relevant more recent literature on the topic, in particular [1, 3]. Given high standards of the journal, I would therefore recommend rejection.

Minor comments

- The manuscript has no abstract so the introduction should explicitly state what the main result of the paper is.
- The inequalities in the definition of chain should be strict.
- It is not clear what *parallel* chains mean. It should be defined first when two chains are parallel.
- The description and notation of zigzag paths in the last paragraph is confusing. I would suggest to describe the zigzag paths first, including specifying on which position the first step is, and then using a shorter notation in the tables without the need for "helper lines" with additional parameter t as an iterator (which is not a parameter of these chains).
- It should be made clear (possibly also in the notation) how the chains are parameterized and why p, q are treated differently to other parameters.
- Symmetric Chain Decomposition (p.1, l.12) does not have to be with capital letters. On the other hand, names of sections (e.g. Section 2) should be. Also titles of all tables should be with capital letters (Parallel chains).
- In the last sum (p.7) it is not clear what each set C_i means.

References

[1] Dhand, V., *Tropical decomposition of Young's partition lattice*, Journal of Algebraic Combinatorics 39(4), 783–806, 2014.

[2] Stanley, R., *Weyl groups, the hard Lefschetz theorem, and the Sperner property* SIAM J. Algebr. Discrete Methods 1, 168–184, 1980.

[3] Zhong, Y., *A Combinatorial Proof of the Unimodality and Symmetry of Weak Composition Rank Sequences*, Annals of Combinatorics, 2022, in press.