Combo Project Seven

Combinatorics (14:332:452)

Project Output Sequences

Team 7

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Basic Sequences:

The following basic sequences use coefficients of 1 and come from applications to chess and lattice walks which we studied over the semester.

f:=1/(1-x-y-x*y)

A001850	The number of ways a King can move across a board in the game of chess.
3, 13, 63, 321, 1683, 8989, 48639, 265729, 1462563, 8097453, 45046719, 251595969, 1409933619, 7923848253, 44642381823, 252055236609, 1425834724419, 8079317057869, 45849429914943, 260543813797441	

f:=1/(1-x-y)

A000984	The number of walks in a 2D lattice with fundamental atomic steps {[0,1], [1,0]}
2, 6, 20, 70, 252, 924, 3432, 12870, 48620, 184756, 705432, 2704156, 10400600, 40116600, 155117520, 601080390, 2333606220, 9075135300, 35345263800, 137846528820	

f:=1/(1-x-y-z)

A006480	The number of walks in a 3D lattice with fundamental atomic steps {[1,0,0], [0,1,0], [0,0,1]}
6, 90, 1680, 34650, 756756, 17153136, 399072960, 9465511770, 227873431500, 5550996791340, 136526995463040, 3384731762521200, 84478098072866400, 2120572665910728000, 53494979785374631680	

f:=1/(1-x-y-xy-z)

A268543	The number of walks in a 3D lattice with fundamental atomic steps {[1,0,0], [0,1,0], [1,1,0], [0,0,1]}
6, 90, 1680, 34650, 756756, 17153136, 399072960, 9465511770, 227873431500, 5550996791340, 136526995463040, 3384731762521200, 84478098072866400, 2120572665910728000, 53494979785374631680	

f:=1/(1-x-y-xy-xz-z) (equivalent to 1/(1-x-y-xy-yz-z))

A268543	The number of walks in a 3D lattice with fundamental atomic steps {[1,0,0], [0,1,0], [1,1,0],[1,0,1], [0,0,1]}
10, 246, 7540, 255430, 9163980, 341237820, 13042646760, 508236930630, 20101587623260, 804500381097556, 32508382071448920, 1324112273705453596, 54296281503438398200, 2239266766596344681400	

f:=1/(1-x-y-z-xy-xz-xyz)

A274780	The number of walks in a 3D lattice with fundamental atomic steps {[1,0,0], [0,1,0], [0,0,1] [1,1,0],[1,0,1],[1,1,1]}
11, 283, 9155, 327811, 12436541, 489807991, 19803209843, 816330309475, 34156900690841, 1446223566321733, 61826502242685653, 2664286789334520559, 115586782462237980905, 5043474229642670729743	

Dominating Sequences - 2D - 1:

The following sequences have a (relatively) large coefficient in one variable and small (1) coefficients in the other variables. We use the form f:=1/(1-a*x-y) and vary in a.

f:=1/(1-2x-y)

A059304	Sequence for generating function of the form 1/(1-a*x-b*y) with a=2, b=1.
4, 24, 160, 1120, 8064, 59136, 439296, 3294720, 24893440, 189190144, 1444724736, 11076222976, 85201715200, 657270374400, 5082890895360, 39392404439040, 305870434467840, 2378992268083200, 18531097667174400	

f:=1/(1-3x-y)

A098658	Sequence for generating function of the form 1/(1-a*x-b*y) with a=3, b=1.
6, 54, 540, 5670, 61236, 673596, 7505784, 84440070, 956987460, 10909657044, 124965162504, 1437099368796, 16581915793800, 191876454185400, 2225766868550640, 25874539846901190	

f:=1/(1-4x-y)

A098430	Sequence for generating function of the form 1/(1-a*x-b*y) with a=4, b=1.
8, 96, 1280, 17920, 258048, 3784704, 56229888, 843448320, 12745441280, 193730707456, 2958796259328, 45368209309696, 697972450918400, 10768717814169600, 166556168859156480	

f:=1/(1-5x-y)

(Not in the OEIS)	Sequence for generating function of the form 1/(1-a*x-b*y) with a=5, b=1.
10, 150, 2500, 43750, 787500, 14437500, 268125000, 5027343750, 94960937500, 1804257812500, 34444921875000, 660194335937500, 12696044921875000, 244852294921875000, 4733811035156250000	

Dominating Sequences - 2D - 2:

The following sequences have a (relatively) large coefficient in one variable and small (1) coefficients in the other variables. We use the form f:=1/(1-a*x-y-x*y) and vary in a.

f:=1/(1-2x-y-xy)

A006442	Sequence for generating function of the form 1/(1-a*x-b*y-c*x*y) with a=2, b=1, c=1. Applications exist to Legendre Polynomial and Lattice paths.
5, 37, 305, 2641, 23525, 213445, 1961825, 18205345, 170195525, 1600472677, 15122515985, 143457011569, 1365435096485, 13033485491077, 124715953657025, 1195966908404545, 11490534389896325	

f:=1/(1-3x-y-xy)

A084768	Sequence for generating function of the form 1/(1-a*x-b*y-c*x*y) with a=3, b=1, c=1. Applications are primarily in Legendre Polynomials (P_n(7)).
7, 73, 847, 10321, 129367, 1651609, 21360031, 278905249, 3668760487, 48543499753, 645382441711, 8614382884849, 115367108888311, 1549456900170553, 20861640747345727, 281483386791966529	

f:=1/(1-4x-y-xy)

A084769	Sequence for generating function of the form 1/(1-a*x-b*y-c*x*y) with a=4, b=1, c=1. Applications are primarily in Legendre Polynomials (P_n(9)).
9, 121, 1809, 28401, 458649, 7544041, 125700129, 2114588641, 35836273449, 610897146201, 10463745263409, 179939616743121, 3104680678772409, 53721299280288201, 931852905510160449	

f:=1/(1-5x-y-xy)

(Not in the OEIS)	Sequence for generating function of the form 1/(1-a*x-b*y-c*x*y) with a=4, b=1, c=1.
11, 181, 3311, 63601, 1256651, 25289461, 515550431, 10611099361, 220016797451, 4588801077301, 96164807352911, 2023268287369681	

Dominating Sequences - 2D - 3:

The following sequences have a (relatively) large coefficient in one variable and small (1) coefficients in the other variables. We use the form f:=1/(1-x-y-a*x*y) and vary in a.

f:=1/(1-x-y-2xy)

A069835	Sequence for generating function of the form 1/(1-a*x-b*y-c*x*y) with a=1, b=1, c=2. Applications exist to Legendre Polynomial and Lattice paths.
4, 22, 136, 886, 5944, 40636, 281488, 1968934, 13875544, 98365972, 700701808, 5011371964, 35961808432, 258805997752, 1867175631136, 13500088649734, 97794850668952, 709626281415076, 5157024231645616	

f:=1/(1-x-y-3xy)

A084771	Sequence for generating function of the form 1/(1-a*x-b*y-c*x*y) with a=1, b=1, c=3. Applications exist to Lattice Paths from (0,0) to (n,0)/(n,n)
5, 33, 245, 1921, 15525, 127905, 1067925, 9004545, 76499525, 653808673, 5614995765, 48416454529, 418895174885, 3634723102113, 31616937184725, 275621102802945, 2407331941640325, 21061836725455905	

f:=1/(1-x-y-4xy)

A069835	Sequence for generating function of the form 1/(1-a*x-b*y-c*x*y) with a=1, b=1, c=4. Coefficients of 1/sqrt(1-12*x+16*x^2)
6, 46, 396, 3606, 33876, 324556, 3151896, 30915046, 305543556, 3038019876, 30354866856, 304523343996, 3065412858696, 30946859111256, 313206733667376, 3176825392214406, 32284147284682596	

f:=1/(1-x-y-5xy)

A098659	Sequence for generating function of the form 1/(1-a*x-b*y-c*x*y) with a=1, b=1, c=5.
7, 61, 595, 6145, 65527, 712909, 7863667, 87615745, 983726695, 11112210781, 126142119187, 1437751935361, 16443380994775, 188609259215725, 2168833084841395, 24994269200292865	

Dominating Sequences - 3D - 1:

The following sequences have a (relatively) large coefficient in one variable and small (1) coefficients in the other variables. We use the form f:=1/(1-a*x-y-*z) and vary in a.

f:=1/(1-x-y-2z)

(Not in the OEIS)	Sequence for generating function of the form 1/(1-a*x-b*y-c*z) with a=1, b=1, c=2.
12, 360, 13440, 554400, 24216192, 1097800704, 51081338880, 2423171013120, 116671196928000, 5684220714332160, 279607286708305920, 13863861299286835200, 692044579412921548800,3474346255828136755200	

f:=1/(1-x-y-3z)

(Not in the OEIS)	Sequence for generating function of the form 1/(1-a*x-b*y-c*z) with a=1, b=1, c=3.
18, 810, 45360, 2806650, 183891708, 12504636144, 872772563520, 62103222722970, 4485232752214500, 327780809531835660, 24185347665291146880, 1798785232606029049200	

f:=1/(1-x-y-4z)

(Not in the OEIS)	Sequence for generating function of the form 1/(1-a*x-b*y-c*z) with a=1, b=1, c=4.
24, 1440, 107520, 8870400, 774918144, 70259245056, 6538411376640, 620331779358720, 59735652827136000, 5820642011476131840, 572635723178610524160, 56786375881878876979200	

f:=1/(1-x-y-5z)

(Not in the OEIS)	Sequence for generating function of the form 1/(1-a*x-b*y-c*z) with a=1, b=1, c=5.
30, 2250, 210000, 21656250, 2364862500, 268017750000, 31177575000000, 3697465535156250, 445065295898437500, 54208953040429687500, 6666357200343750000000, 826350527959277343750000	

Additive Relationships - 2D - 1:

The following sequences add together sequences of the form 1/(1-a*x-b*y) to explore their additive relationship.

f:=1/(1-x-y) + 1/(1-x-y)

A028329	Sequence for the generating function of f:=1/(1-x-y) + 1/(1-x-y). Applications include the twice central binomial coefficients and the lattice walks from (n-1,n-1) to (n,n)
4, 12, 40, 140, 504, 1848, 6864, 25740, 97240, 369512, 1410864, 5408312, 20801200, 80233200, 310235040, 1202160780, 4667212440, 18150270600, 70690527600, 275693057640	

f:=1/(1-x-y) + 1/(1-2x-y)

(Not in the OEIS)	Sequence for the generating function of f:=1/(1-x-y) + 1/(1-2x-y).
6, 30, 180, 1190, 8316, 60060, 442728, 3307590, 24942060, 189374900, 1445430168, 11078927132, 85212115800, 657310491000, 5083046012880, 39393005519430, 305872768074060, 2379001343218500	

f:=1/(1-x-y) + 1/(1-3x-y)

(Not in the OEIS)	Sequence for the generating function of f:=1/(1-x-y) + 1/(1-3x-y).
8, 60, 560, 5740, 61488, 674520, 7509216, 84452940, 957036080, 10909841800, 124965867936, 1437102072952, 16581926194400, 191876494302000, 2225767023668160, 25874540447981580	

Additive Relationships - 2D - 2:

The following sequences add together sequences of the form 1/(1-a*x-b*y-c*x*y) to explore their additive relationship.

f:=1/(1-x-y-xy) + 1/(1-x-y-xy)

(Not in the OEIS)	Sequence for the generating function of f:=1/(1-x-y-xy) + 1/(1-x-y-xy).
6, 26, 126, 642, 3366, 17978, 97278, 531458, 2925126, 16194906, 90093438, 503191938, 2819867238, 15847696506, 89284763646, 504110473218, 2851669448838, 16158634115738	

f:=1/(1-x-y-xy) + 1/(1-x-y-2xy)

(Not in the OEIS)	Sequence for the generating function of f:=1/(1-x-y-xy) + 1/(1-x-y-2xy).
7, 35, 199, 1207, 7627, 49625, 330127, 2234663, 15338107, 106463425, 745748527, 5262967933, 37371742051, 266729846005, 1911818012959, 13752143886343, 99220685393371, 717705598472945	

f:=1/(1-x-y-xy) + 1/(1-x-y-3xy)

(Not in the OEIS)	Sequence for the generating function of f:=1/(1-x-y-xy) + 1/(1-x-y-3xy).
8, 46, 308, 2242, 17208, 136894, 1116564, 9270274, 77962088, 661906126, 5660042484, 48668050498, 420305108504, 3642646950366, 31661579566548, 275873158039554, 2408757776364744, 21069916042513774	

Additive Relationships - 3D:

The following sequences add together sequences of the form 1/(1-a*x-b*y-c*z) to explore their additive relationship.

f:=1/(1-x-y-z) + 1/(1-x-y-z)

(Not in the OEIS)	Sequence for the generating function of f:=1/(1-x-y-z) + 1/(1-x-y-z).
12, 180, 3360, 69300, 1513512, 34306272, 798145920, 18931023540, 455746863000, 11101993582680, 273053990926080, 6769463525042400, 168956196145732800, 4241145331821456000	

f:=1/(1-x-y-z) + 1/(1-x-y-2z)

(Not in the OEIS)	Sequence for the generating function of f:=1/(1-x-y-z) + 1/(1-x-y-2z).
18, 450, 15120, 589050, 24972948, 1114953840, 51480411840, 2432636524890, 116899070359500, 5689771711123500, 279743813703768960, 13867246031049356400, 692129057510994415200	

Subtractive Relationships - 3D:

The following sequences add together sequences of the form 1/(1-a*x-b*y-c*z) to explore their additive relationship.

f:=1/(1-x-y-2z) - 1/(1-x-y-z)

(Not in the OEIS)	Sequence for the generating function of f:=1/(1-x-y-2z) - 1/(1-x-y-z).
6, 270, 11760, 519750, 23459436, 1080647568, 50682265920, 2413705501350, 116443323496500, 5678669717540820, 279470759712842880, 13860476567524314000	

f:=1/(1-x-y-3z) - 1/(1-x-y-z)

(Not in the OEIS)	Sequence for the generating function of f:=1/(1-x-y-3z) - 1/(1-x-y-z).

12, 720, 43680, 2772000, 183134952, 12487483008, 872373490560, 62093757211200, 4485004878783000, 327775258535044320, 24185211138295683840, 1798781847874266528000

f:=1/(1-x-y-3z) - 1/(1-x-y-2z)

(Not in the OEIS)	Sequence for the generating function of f:=1/(1-x-y-3z) - 1/(1-x-y-z).
6, 450, 31920, 2252250, 159675516, 11406835440, 821691224640, 59680051709850, 4368561555286500, 322096588817503500, 23905740378582840960, 1784921371306742214000, 133993330174413655898400	

Multiplicative Relationships - 2D:

The following sequences multiply together sequences of the form 1/(1-a*x-b*y-c*x*y) to explore their multiplicative relationship.

f:=1/(1-x-y-xy) * 1/(1-x-y-xy)

A243949	Sequence for the generating function of f:=1/(1-x-y-xy) * 1/(1-x-y-xy). Applications are the central Delannoy numbers.
9, 169, 3969, 103041, 2832489, 80802121, 2365752321, 70611901441, 2139090528969, 65568745087209, 2029206892664961, 63300531617048961, 1987912809986437161	

f:=1/(1-x-y-xy) * 1/(1-x-y-2xy)

(Not in the OEIS)	Sequence for the generating function of f:=1/(1-x-y-xy) * 1/(1-x-y-xy).
12, 286, 8568, 284406, 10003752, 365277004, 13691294832, 523202862886, 20293857259272, 796513835069316, 31564317447767952, 1260840985302013116, 50703762708314475408, 2050739453153107127256	

f:=1/(1-x-y) * 1/(1-x-y)

A002894	Sequence for the generating function of f:=1/(1-x-y) * 1/(1-x-y). Applications are the sequence a(n)=binomial(2n,n)^2.
4, 36, 400, 4900, 63504, 853776, 11778624, 165636900, 2363904400, 34134779536, 497634306624, 7312459672336, 108172480360000, 1609341595560000, 24061445010950400, 361297635242552100	

f:=1/(1-x-y) * 1/(1-2x-y)

(Not in the OEIS)	Sequence for the generating function of f:=1/(1-2x-y) * 1/(1-x-y).
8, 144, 3200, 78400, 2032128, 54641664, 1507663872, 42403046400, 1210319052800, 34954014244864, 1019155059965952, 29951834817888256, 886148959109120000, 26367452701655040000	

Multiplicative Relationships - 3D:

The following sequences multiply together sequences of the form 1/(1-a*x-b*y-c*z) to explore their multiplicative relationship.

f:=1/(1-x-y-z) * 1/(1-x-y-z)

A268553	Sequence for the generating function of f:=1/(1-x-y-z) * 1/(1-x-y-z). Application is the diagonal of the rational function 1/((1-uv-uw-vw)*(1-xy-xz-yz))
36, 8100, 2822400, 1200622500, 572679643536, 294230074634496, 159259227403161600, 89595913068008532900, 51926300783585192250000, 30813565377466975498995600	

f:=1/(1-x-y-z) * 1/(1-x-y-2z)

(Not in the OEIS)	Sequence for the generating function of f:=1/(1-x-y-z) * 1/(1-x-y-z).
72, 32400, 22579200, 19209960000, 18325748593152, 18830724776607744, 20385181107604684800, 22936553745410184422400, 26586266001195618432000000, 31553090946526182910971494400	

K Components Relationships - 2D:

The following sequences are the result of taking a generating function of the form f:=1/(1-a*x-b*y-c*x*y) and finding the sequence of the K-components generating function: f^k/k!.

f:=(1/(1-x-y))^2/2!

A033876	Sequence for the generating function of f:=1/(1-x-y)^2/2!. Applications are the number of edges in an odd graph and the number of maximal cliques in the (n+2)-odd graph
3, 15, 70, 315, 1386, 6006, 25740, 109395, 461890, 1939938, 8112468, 33801950, 140408100, 581690700, 2404321560, 9917826435, 40838108850, 167890003050	

f:=(1/(1-2x-y))^2/2!

(Not in the OEIS)	Sequence for the generating function of f:=1/(1-2x-y)^2/2!.
6, 60, 560, 5040, 44352, 384384, 3294720, 28005120, 236487680, 1986496512, 16614334464, 138452787200, 1150223155200, 9530420428800, 78784808878080	

f:=(1/(1-3x-y))^2/2!

(Not in the OEIS)	Sequence for the generating function of f:=1/(1-3x-y)^2/2!.
9, 135, 1890, 25515, 336798, 4378374, 56293380, 717740595, 9091380870, 114551398962, 1437099368796, 17963742109950, 223855863216300, 2782208585688300, 34499386462534920	

K Components Relationships - 3D:

The following sequences are the result of taking a generating function of the form f:=1/(1-a*x-b*y-c*z) and finding the sequence of the K-components generating function: f^k/k!.

f:=(1/(1-x-y-z))^2/2!

(Not in the OEIS)	Sequence for the generating function of f:=1/(1-x-y-z)^2/2!.
12, 315, 8400, 225225, 6054048, 162954792, 4389802560, 118318897125, 3190228041000, 86040450265770, 2320958922871680, 62617537606642200, 1689561961457328000	

f:=(1/(1-x-y-2*z))^2/2!

(Not in the OEIS)	Sequence for the generating function of f:=1/(1-x-y-2z)^2/2!.
24, 1260, 67200, 3603600, 193729536, 10429106688, 561894727680, 30289637664000, 1633396756992000, 88105421072148480, 4753323874041200640, 256481434036806451200, 13840891588258430976000	