1. (15 points) Solve explicitly the recurrence equation

$$x_n = 4x_{n-2} + 1 \quad ,$$

with initial conditions

$$x_0 = 1, x_1 = 2 \quad .$$

2. (15 points) In a certain species of animals, only one-year-old, two-year-old are fertile. The probabilities of a one-year-old, two-year-old, female to give birth to a new female are $\frac{1}{2}$, $\frac{2}{3}$, respectivley.
Assuming that there were 10 females born at $n = 0$, 12 females born at $n = 1$. How many females were born at $n = 5$?
3. (15 points) Write the Maple command to solve the following recurrence (Do not solve it!)

$$a(n) = 10a(n-1) - 21a(n-2) + n^3 \quad , \quad a(0) = 5 \quad , \quad a(1) = 3$$

4. (15 points) Solve the following initial value problem

$$x''(t) + x(t) = 0 \quad , \quad x(0) = 1 \quad , \quad x'(0) = 0$$

6. (15 points) Write the Maple command to solve the following intial value problem. Do not solve it

$$y^{(5)}(t)+y^{(3)}(t)+y(t) = \sin t \quad , \quad y^{(4)}(0) = 1, y^{(3)}(0) = 0, y^{(2)}(0) = -1, y'(0) = 1, y(0) = 3.$$

7. (15 points) Solve the recurrence initial value problem

$$a(n) = a(n-5) - a(n-6) \quad ,$$

$$a(0) = 0, a(1) = 0, a(2) = 0, a(3) = 0, a(4) = 0, a(5) = 0 \quad .$$

8. (15 points) (a) For which values of k is $x = 1$ a stable steady-state of the discrete continuous dynamiical system

$$x(n) = x(n-1) + \frac{x(n-1)(1 - x(n-1))(2 - x(n-1))}{k}$$

9. (15 points) You enter a fair casino (where the prob. of winning a dollar is 0.48 and the prob. of losing a dollar is 0.52), and you must exit if you are broke or you have 200 dollars. If right now you have 150 dollars, how likely are you to exit a winner?
10. (15 points) Find the equilibrium point(s) of the continuous dynamical system

$$x'(t) = x(t) - y(t) \quad , \quad y'(t) = -x(t) + y(t) \quad .$$

Is it stable, semi-stable, or not stable? Explain!

11. (15 points) Find all the steady-states of the discrete dynamical system

$$a_1(n+1) = \frac{a_1(n)}{2 - a_2(n)}$$

$$a_2(n+1) = \frac{a_2(n)}{2 - a_3(n)}$$

$$a_3(n+1) = \frac{a_3(n)}{4 - a_1(n)}$$

12. (15 points) Find all the stable steady-states of the discrete dynamical system

$$a_1(n+1) = \frac{a_1(n)}{2 - a_2(n)}$$

$$a_2(n+1) = \frac{a_2(n)}{2 - a_3(n)}$$

$$a_3(n+1) = \frac{a_3(n)}{4 - a_1(n)}$$

13. (20 points) Recall that, in the Hardy-Weinberg model, if the fraction of the population of AA is u, the fraction of Aa is v (and hence the fraction of aa is $1 - u - v$) then in the next generation it is

$$[u, v] \to [u^2 + v\,u + \frac{v^2}{4}, vu + 2u(1 - u - v) + \frac{v^2}{2} + v\,(1 - u - v)] \quad .$$

If in this generation $\frac{1}{4}$ of the population is AA, $\frac{2}{5}$ and Aa. What fraction of the population will be aa in 100 generations?