NAME: (print!) ____

E-Mail address: ____________________

MATH 336 (1), Dr. Z. , Final Exam, Tue., Dec.. 16, 2025, 8:00-11:00 am, BECK 003

FRAME YOUR FINAL ANSWER(S) TO EACH PROBLEM
No Calculators! No books! No Notes! To ensure maximum credit, organize your work neatly and be sure to show all your work.
Do not write below this line

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1. (out of 15)

2. (out of 15)

3. (out of 15)

4. (out of 15)

5. (out of 15)

6. (out of 15)

7. (out of 15)

8. (out of 15)

9. (out of 15)

10. (out of 15)

11. (out of 15)

12. (out of 15)

13. (out of 20)

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tot.: (out of 200)

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1. (15 points) Solve explicitly the recurrence equation

$$x_n = 4x_{n-2} + 1 \quad ,$$

with initial conditions

$$x_0 = 1, x_1 = 2 \quad .$$

2. (15 points) In a certain species of animals, only one-year-old, two-year-old are fertile. The probabilities of a one-year-old, two-year-old, female to give birth to a new female are $\frac{1}{2}$, $\frac{2}{3}$, respectivley.
Assuming that there were 10 females born at $n = 0$, 12 females born at $n = 1$. How many females were born at $n = 5$?

3. (15 points) Write the Maple command to solve the following recurrence (Do not solve it!)

$$a(n) = 10a(n-1) - 21a(n-2) + n^3 \quad , \quad a(0) = 5 \quad , \quad a(1) = 3$$

4. (15 points) Solve the following initial value problem

$$x''(t) + x(t) = 0 \quad , \quad x(0) = 1 \quad , \quad x'(0) = 0$$

6. (15 points) Write the Maple command to solve the following intial value problem. Do not solve it

$$y^{(5)}(t)+y^{(3)}(t)+y(t)=\sin t \quad , \quad y^{(4)}(0)=1, y^{(3)}(0)=0, y^{(2)}(0)=-1, y'(0)=1, y(0)=3.$$

7. (15 points) Solve the recurrence initial value problem

$$a(n)=a(n-5)-a(n-6) \quad ,$$

$$a(0)=0, a(1)=0, a(2)=0, a(3)=0, a(4)=0, a(5)=0 \quad .$$

8. (15 points) (a) For which values of k is $x=1$ a stable steady-state of the discrete continuous dynamiical system

$$x(n)=x(n-1)+\frac{x(n-1)(1-x(n-1))(2-x(n-1))}{k}$$

9. (15 points) You enter a fair casino (where the prob. of winning a dollar is 0.48 and the prob. of losing a dollar is 0.52), and you must exit if you are broke or you have 200 dollars. If right now you have 150 dollars, how likely are you to exit a winner?

10. (15 points) Find the equilibrium point(s) of the continuous dynamical system

$$x'(t) = x(t) - y(t) \quad , \quad y'(t) = -x(t) + y(t) \quad .$$

Is it stable, semi-stable, or not stable? Explain!

11. (15 points) Find all the steady-states of the discrete dynamical system

$$a_1(n+1) = \frac{a_1(n)}{2 - a_2(n)}$$

$$a_2(n+1) = \frac{a_2(n)}{2 - a_3(n)}$$

$$a_3(n+1) = \frac{a_3(n)}{4 - a_1(n)}$$

12. (15 points) Find all the stable steady-states of the discrete dynamical system

$$a_1(n+1) = \frac{a_1(n)}{2 - a_2(n)}$$

$$a_2(n+1) = \frac{a_2(n)}{2 - a_3(n)}$$

$$a_3(n+1) = \frac{a_3(n)}{4 - a_1(n)}$$

13. (20 points) Recall that, in the Hardy-Weinberg model, if the fraction of the population of AA is u, the fraction of Aa is v (and hence the fraction of aa is $1 - u - v$) then in the next generation it is

$$[u, v] \to [u^2 + v\,u + \frac{v^2}{4}, vu + 2u(1 - u - v) + \frac{v^2}{2} + v\,(1 - u - v)] \quad .$$

If in this generation $\frac{1}{4}$ of the population is AA, $\frac{2}{5}$ and Aa. What fraction of the population will be aa in 100 generations?