NAME: (print!)

E-Mail address: ____________________

MATH 336 (1), Dr. Z. , Make up Midterm Exam , Fri., Dec. 5, 2025, 2:45-4:05 pm on-line

Please email completed exam to
ShaloshBEkhad@gmail.com
FRAME YOUR FINAL ANSWER(S) TO EACH PROBLEM
No Calculators! No books! No Notes! To ensure maximum credit, organize your work neatly and be sure to show all your work.
Do not write below this line

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1. (out of 10)

2. (out of 10)

3. (out of 10)

4. (out of 10)

5. (out of 10)

6. (out of 10)

7. (out of 10)

8. (out of 10)

9. (out of 20)

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tot.: (out of 100)

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1. (10 points) Solve explicitly the recurrence equation

$$x_n = x_{n-1} + 6x_{n-2} \quad ,$$

with initial conditions

$$x_0 = 2, x_1 = 1 \quad .$$

2. (10 points) In a certain species of animals, only one-year-old, two-year-old and three-year olds are fertile. The probabilities of a one-year-old, two-year-old, and three-year female to give birth to a new female are p_1, p_2, p_3 respectivley.
Assuming that there were c_0 females born at $n = 0$, c_1 females born at $n = 1$ and c_2 females born at $n = 2$, Set up a recurrence that will enable you to find the **expected** number of females born at time n.
In terms of $c_0, c_1, c_2, p_1, p_2, p_3$, how many females were born at $n = 3$?

3. (10 points) Prove that $a_1(n) = 2^{5^n}$ satisfies the non-linear recurrence equation

$$a(n) = a(n-1)^5 \quad .$$

Is the following constant multiple of the seqeuncec $a_1(n)$, given by $a_2(n) = 3 \cdot 2^{3^n}$, also a solution? Why?

4. (10 points) Solve the following recurrence subject to the initla conditions.

$$a(n) = 5a(n-1) - 6a(n-2) + 1 \quad ; \quad a(0) = 2 \quad , \quad a(1) = 3 \quad .$$

5. (10 points) (a) Find all the steady-states of the non-linear recurrence

$$x(n) = kx(n-1)(1 - x(n-1)) \quad ,$$

where k is a positive number between 0 and 4. Express your answer in terms of k (if needed).

(b) For which values of k is $x = 0$ the only stable steady-state? For which values of k is the other steady-state stable? For which values of k neither of them are?

6. (10 points) Show that $x = 4$ is a steady-state of the discrete dynamical system, for any real number k,

$$x(n) = x(n-1) - \frac{(1 - x(n-1))\,(4 - x(n-1))}{k} \quad ,$$

For what values of the parameter, k, is $x = 4$ a stable steady-state?

7. (10 points) You enter a fair casino (where the prob. of winning a dollar is $\frac{1}{2}$ and the prob. of losing a dollar is $\frac{1}{2}$), and you must exit if you are broke or you have 200 dollars. If right now you have 150 dollars, how likely are you to exit a winner? How long would you expect to be in the casino until you leave either a winner or loser?

8. (10 points) Find the equilibrium point(s) of the continuous dynamical system

$$x'(t) = 7x(t) - 3y(t) \quad , \quad y'(t) = 3x(t) - 7y(t) \quad .$$

Is it stable, semi-stable, or not stable? Explain!

9. (20 points) (a) (10 points) Find all the steady-states of the discrete dynamical system

$$a_1(n+1) = \frac{a_1(n)}{4 - a_2(n)}$$

$$a_2(n+1) = \frac{a_2(n)}{5 - a_1(n)}$$

b (10 points) For these steady-states, which of them are stable? unstable? semi-stable? Explain.