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Homework #9

1.) $a(n+2) = 5a(n+1) - 6a(n)$, $a(0) = 1$ $a(L) = 2$

$z^2 - 5z + 6 = 0 \Rightarrow z = 2, 3$

$a(n) = c_1 2^n + c_2 3^n$

$a(0) = c_1 + c_2 = 1 \Rightarrow c_2 = 1 - c_1$ and $c_1 = 1 - c_2$

$a(L) = c_1 2^L + c_2 3^L = c_1 2^L + (1 - c_1)3^L = c_1 2^L + 3^L - c_1 3^L$

$= c_1(2^L - 3^L) + 3^L = 2 \Rightarrow c_1 = \frac{2 - 3^L}{2^L - 3^L}, c_2 = 1 - \frac{2-3^L}{2^L - 3^L}$

$a(n) = \left(\frac{2-3^L}{2^L-3^L}\right)2^n + \left(1 - \frac{2-3^L}{2^L-3^L}\right)3^n$

2.) $n = 700$, $L = 1{,}000$

$a(n) = pa(n+1) + qa(n-1)$

i) $p = 0.5$, $q = 0.5$

$a(n) = n/L = 700/1000 = 0.7$

i') $b(n) = n(L-n) = 700(1{,}000 - 700) = 210{,}000$

ii) $p = 0.49$, $q = 0.51$

$a(n) = \frac{1 - (q/p)^n}{1 - (q/p)^L} = \frac{1 - (.51/.49)^{700}}{1 - (.51/.49)^{1{,}000}} = 0.00000613$

iii) $p = 0.499$, $q = 0.501$

$a(n) = \frac{1 - (.501/.499)^{700}}{1 - (.501/.499)^{1000}} = 0.288$

3.) $a(n) = pa(n+1) + qa(n-1),\ a(0) = 0 \quad a(L) = 1$

$pa(n+2) - a(n+1) + qa(n) = 0$

$z^2 - \frac{1}{p} z + \frac{q}{p} = 0 \Rightarrow z = 1, \frac{q}{p}$

$a(n) = c_1 1^n + c_2 (q/p)^n$

$a(0) = c_1 + c_2 = 0 \Rightarrow c_2 = -c_1$

$a(L) = c_1 + c_2 (q/p)^L = c_1 - c_1 (q/p)^L = c_1(1-(q/p)^L) = 1$

$c_1 = \frac{1}{1-(q/p)^L} \qquad c_2 = \frac{-1}{1-(q/p)^L}$

$a(n) = \frac{1}{1-(q/p)^L} - \frac{(q/p)^n}{1-(q/p)^L} = \frac{1-(q/p)^n}{1-(q/p)^L}$

4.) $b(n) = \frac{1}{2} b(n+1) + \frac{1}{2} b(n-1) + 1,\ b(0) = 0,\ b(L) = 0$

$b(n+1) - 2b(n) + b(n-1) = -2$

$z^2 - 2z + 1 = 0 \Rightarrow z = 1, 1$

$b_{homo} = c_1 1^n + c_2 n 1^n = c_1 + c_2 n$

$b_{part} \neq c_3 + c_4 n$ since that is already the homo sol.

$\Rightarrow b_{part} = c_3 n^2 \Rightarrow$ plug into recurrence

$-\frac{1}{2} c_3 (n+1)^2 + c_3 n^2 - \frac{1}{2} c_3 (n-1)^2 = 1$

$c_3(-\frac{1}{2}n^2 - n - \frac{1}{2} + n^2 - \frac{1}{2}n^2 + n - \frac{1}{2}) = 1 \Rightarrow (-1)c_3 = 1,\ c_3 = -1$

$b_{part} = -n^2$

$b_{gen} = b_{homo} + b_{part} = c_1 + c_2 n - n^2$

$b(0) = c_1 = 0$

$b(L) = c_1 + c_2 L - L^2 = 0 \Rightarrow c_2 L = L^2 \Rightarrow c_2 = L$

$b(n) = Ln - n^2 = n(L-n)$