HWQ #1

$a(n+2) = 5a(n+1) - 6a(n) = 0$

$a(n) = 1 \quad a(1) = 2$

$z^2 = 5z - 6$

$z^2 - 5z + 6 = 0$

$(z-2)(z-3) = 0$

$z = 2, 3$

$a(n) = A_1 \cdot 2^n + A_2 \cdot 3^n$

$0 = A_1 + A_2$

$2 = A_1 \cdot 2^1 + A_2 \cdot 3^1$

$2 = A_1(2^1 - 3^1)$

$A_1 = \frac{2}{2^1 - 3^1} \qquad A_2 = \frac{-2}{2^1 - 3^1}$

$a(n) = \frac{2(2^n)}{2^1 - 3^1} - \frac{2(3^n)}{2^1 - 3^1}$

2i) $\frac{n}{L} = \frac{700}{1000} = \frac{7}{10}$

i') $n(L-n) = 700(1000-700) = 210{,}000$

ii) $\frac{1-\left(\frac{0.51}{0.49}\right)^{700}}{1-\left(\frac{0.51}{0.49}\right)^{1000}} = 6.13 \times 10^{-6}$

iii) $\frac{1-\left(\frac{0.501}{0.499}\right)^{700}}{1-\left(\frac{0.501}{0.499}\right)^{1000}} = 0.288$

#3

$a(n) = P(\text{win} \mid \text{position} = n)$

$a(n) = p\,a(n+1) + (1-p)\,a(n-1)$

$a(n+1) = p\,a(n+2) + (1-p)\,a(n)$

$$a(n+2) = \frac{a(n+1)}{p} - \frac{(1-p)\,a(n)}{p}$$

$$z^2 = \left(\frac{1}{p}\right)z - \left(\frac{1-p}{p}\right)$$

$$z^2 - \left(\frac{1}{p}\right)z + \left(\frac{1-p}{p}\right) = 0$$

$$(z-1)\left(z - \left(\frac{1-p}{p}\right)\right) = 0$$

$$z = 1, \left(\frac{1-p}{p}\right)$$

$$a(n) = A_1 + A_2\left(\frac{1-p}{p}\right)^n$$

$$a(0) = A_1 + A_2 = 0$$

$$a(L) = A_1 + A_2\left(\frac{1-p}{p}\right)^L = 1$$

$$a(L) = A_1\left(1 - \left(\frac{1-p}{p}\right)^L\right) = 1$$

$$A_1 = \frac{1}{1-\left(\frac{1-p}{p}\right)^L} \qquad A_2 = \frac{-1}{1-\left(\frac{1-p}{p}\right)^L}$$

$$a(n) = \frac{1}{1-\left(\frac{1-p}{p}\right)^L} - \frac{\left(\frac{1-p}{p}\right)^n}{1-\left(\frac{1-p}{p}\right)^L}$$

$$a(n) = \frac{1-\left(\frac{1-p}{p}\right)^n}{1-\left(\frac{1-p}{p}\right)^L}$$

#4

$a(n) = \frac{1}{2}a(n-1) + \frac{1}{2}a(n+1) + 1$ (inhomogeneous)

$a(0) = 0$, $a(L) = 0$

$a(n+1) = \frac{1}{2}a(n) + \frac{1}{2}a(n+2) + 1$

$a(n+2) - 2a(n+1) + a(n) = -2$

First solve

$a(n+2) - 2a(n+1) + a(n) = 0$

$z^2 - 2z + 1 = 0$

$z = 1$ (mult. 2)

$a(n) = A_1(1)^n + A_2(1)^n n$

$a(n) = A_1 + A_2 n$

Particular soln $a(n) = \alpha n + b$

$\alpha(n+2) + b - 2(\alpha(n+1) + b) + \alpha n + b = -1$

$\alpha n + 2\alpha + b - 2\alpha n - 2\alpha - 2b + \alpha n + b = -1$

Particular soln $a(n) = \alpha n^2 + bn + c$

$\alpha(n+2)^2 + b(n+2) + c$

$-2\alpha(n+1)^2 - 2b(n+1) - 2c$

$+\alpha n^2 + bn + c$

$= \alpha n^2 + 4\alpha n + 4\alpha + bn + 2b$

$-2\alpha n^2 - 4\alpha n - 2\alpha - 2bn - 2b$

$+\alpha n^2 + bn$

$= 2\alpha = -2$

$\Rightarrow \alpha = -1$, $a(n) = -n^2$

$a(n) = A_1 + A_2 n + -n^2$

$a(0) = A_1 = 0$

$a(L) = A_1 + A_2 L - L^2$

$A_2 = L$

$a(n) = Ln - n^2 \Rightarrow \boxed{a(n) = n(L-n)}$