

HW 9

① $z^2 - 5z + 6 = 0 \Rightarrow (z-3)(z-2) = 0$ $a(n) = A_1 3^n + A_2 2^n$

$a(0) = A_1 + A_2 = 1 \rightarrow A_2 = 1 - A_1$

$a(1) = A_1 3^1 + A_2 2^1 = 2 \Rightarrow A_1 3^1 + (1 - A_1) 2^1 = 2$

$A_1(3^1 - 2^1) = 2 - 2^1 \Rightarrow A_1 = \frac{2 - 2^1}{3^1 - 2^1}$ $A_2 = 1 - \frac{2 - 2^1}{3^1 - 2^1}$

$a(n) = \left(\frac{2 - 2^1}{3^1 - 2^1} \right) 3^n + \left(1 - \frac{2 - 2^1}{3^1 - 2^1} \right) 2^n$

② i) $\frac{700}{1000} = 0.7$ i') $700 \times 300 = \boxed{210,000}$

ii) $\frac{q}{p} = 1.04$ $a(n) = .49a(n+1) + .51a(n-1)$

$.49z^2 - z + .51 = 0 \Rightarrow (z-1)(.49z - .51) = 0$

$z_1 = 1$ $z_2 = 1.04$ $a(n) = A_1 + A_2 1.04^n$

$a(0) = A_1 + A_2$ $a(1) = A_1 + A_2(1.04)^1 = 1$

$A_1(1 - 1.04^1) = 1$ $A_1 = \frac{1}{1 - 1.04^1}$ $A_2 = \frac{-1}{1 - 1.04^1}$

$a(n) = \frac{1 - (1.04)^n}{1 - 1.04}$ $\frac{1 - (1.04)^{700}}{1 - 1.04^{1000}} = \boxed{6.14 \times 10^{-6}}$

iii) $\frac{1 - \left(\frac{.501}{.499}\right)^{700} - 1.04^1}{1 - \left(\frac{.501}{.499}\right)^{1000}} = \boxed{.301}$

③ $a(n) = pa(n+1) + (1-p)a(n-1) \Rightarrow pz^2 - z + (1-p) = 0 \Rightarrow (z-1)(pz + p - 1) = 0$

$z_1 = 1$ $z_2 = \frac{1-p}{p}$ $a(n) = A_1 + A_2 \left(\frac{1-p}{p}\right)^n$

$a(0) = A_1 + A_2$ $a(1) = A_1 + A_2 \left(\frac{1-p}{p}\right)^1 = 1 \Rightarrow A_1 \left(1 - \left(\frac{1-p}{p}\right)^1\right) = 1$

$A_1 = \frac{1}{1 - \left(\frac{1-p}{p}\right)^1}$ $A_2 = \frac{-1}{1 - \left(\frac{1-p}{p}\right)^1}$

$a(n) = \frac{1 - \left(\frac{1-p}{p}\right)^n}{1 - \left(\frac{1-p}{p}\right)^1}$ where $q = 1-p$

④ Recurrence $A(n) = 1 + \frac{1}{2}A(n+1) + \frac{1}{2}A(n-1)$, $A(0) = 0$ $A(1) = 0$

$A(n) = A + B_n - n^2$ $A(0) = A = 0$ $A(n) = n(1-n)$