Homework for Lecture 8 of Dr. Z.'s Dynamical Models in Biology class

Email the answers (as .pdf file) to

ShaloshBEkhad@gmail.com

by 8:00pm Monday, Oct. 6,, 2021.

Subject: hw8

with an attachment hw8FirstLast.pdf and/or hw8FirstLast.txt

Using

`http://sites.math.rutgers.edu/~zeilberg/Bio25/DMB8.txt`

1. Find ten random rational functions using `raRF(x,3)` .

For each of them find all the stable steady-states.

Then using `Orb(f,x,x0,2000,2020);` for `x0=-100.1, -10.1, 10.1, 100.1`

verify that the orbit always converges to that stable steady-state.

2. Find the terms between 2000-th and 2030-th terms of the **orbit** starting at $x_0 = 0.5$ of the recurrence

$$x_n = kx_{n-1}(1 - x_{n-1}) \quad ,$$

for $k = 1$, $k = 2$, $k = 2.5$, $k = 3.1$, $k = 3.2$, $k = 3.3$, $k = 3.5$. What do you find?

3. By Playing "High-Low" find the smallest positive integer such that `k`,

```
f:=MakeNicePol([1,2,3,4,5,6,7,8],k,x);
```

has two stable steady-states. Pick random `x0` from their neighborhood and verify that indeed the orbits converges to them.

Then keep upping k to find the smallest integer such that it has four stable steady-states. Pick random `x0` from their neighborhood and verify that indeed the orbits converges to them.

1. Find ten random rational functions using `raRF(x,3)` .

For each of them find all the stable steady-states.

Then using `Orb(f,x,x0,2000,2020);` for `x0=-100.1, -10.1, 10.1, 100.1`

verify that the orbit always converges to that stable steady-state.

1

```
> read ("DMB8.txt")
```

For a list of the Main procedures type: Help8(); for help with a specific procedure type: Help8(ProcedureName); for example Help8(RandomMC);
For a list of the other procedures type: Help8a(); for help with a specific procedure type: Help8(ProcedureName); for example Help8(RandomMC);

```
> Help8( );
```

The available procedures are: Orb, SS, SSS

```
> Help8a( );
```

The other procedures are: MakeNicePol, raRF

```
> f := raRF(x, 3);
```

$$f := \frac{9x^3 + 2x^2 + 4x + 8}{4x^3 + 7x^2 + 9x + 5}$$

```
> SSS(f, x);
  evalf(%);
```

$$[RootOf(4\_Z^4 - 2\_Z^3 + 7\_Z^2 + \_Z - 8, index = 1)]$$
$$[0.9127501374]$$

```
> Orb(f, x, -100.1, 2000, 2020);
[0.9127501374, 0.9127501374, 0.9127501374, 0.9127501374, 0.9127501374, 0.9127501374, 0.9127501374, 0.9127501374, 0.9127501374, 0.9127501374, 0.9127501374,
  0.9127501374, 0.9127501374, 0.9127501374, 0.9127501374, 0.9127501374, 0.9127501374, 0.9127501374, 0.9127501374, 0.9127501374]
> Orb(f, x, -10.1, 2000, 2020);
[0.9127501374, 0.9127501374, 0.9127501374, 0.9127501374, 0.9127501374, 0.9127501374, 0.9127501374, 0.9127501374, 0.9127501374, 0.9127501374, 0.9127501374,
  0.9127501374, 0.9127501374, 0.9127501374, 0.9127501374, 0.9127501374, 0.9127501374, 0.9127501374, 0.9127501374, 0.9127501374]
> Orb(f, x, 10.1, 2000, 2020);
[0.9127501374, 0.9127501374, 0.9127501374, 0.9127501374, 0.9127501374, 0.9127501374, 0.9127501374, 0.9127501374, 0.9127501374, 0.9127501374, 0.9127501374,
  0.9127501374, 0.9127501374, 0.9127501374, 0.9127501374, 0.9127501374, 0.9127501374, 0.9127501374, 0.9127501374, 0.9127501374]
> Orb(f, x, 100.1, 2000, 2020);
[0.9127501369, 0.9127501369, 0.9127501369, 0.9127501369, 0.9127501369, 0.9127501369, 0.9127501369, 0.9127501369, 0.9127501369, 0.9127501369, 0.9127501369,
  0.9127501369, 0.9127501369, 0.9127501369, 0.9127501369, 0.9127501369, 0.9127501369, 0.9127501369, 0.9127501369, 0.9127501369]
>
```

can clearly see the values converge

2

```
> read ("DMB8.txt")
```

For a list of the Main procedures type: Help8(); for help with a specific procedure type: Help8(ProcedureName); for example Help8(RandomMC);
For a list of the other procedures type: Help8a(); for help with a specific procedure type: Help8(ProcedureName); for example Help8(RandomMC);

```
> Help8( );
```

The available procedures are: Orb, SS, SSS

```
> Help8a( );
```

The other procedures are: MakeNicePol, raRF

```
> f := raRF(x, 3);
```

$$f := \frac{3x^3 + 5x^2 + 3x + 3}{x^3 + 3x^2 + 10x + 8}$$

```
> SSS(f, x);
  evalf(%);
```

$$[RootOf(\_Z^4 + 5\_Z^2 + 5\_Z - 3, index = 1)]$$
$$[0.4186175181]$$

```
> Orb(f, x, -100.1, 2000, 2020);
[0.4186175182, 0.4186175181, 0.4186175180, 0.4186175179, 0.4186175182, 0.4186175181, 0.4186175180, 0.4186175179, 0.4186175182, 0.4186175181, 0.4186175180, 0.4186175179,
  0.4186175182, 0.4186175181, 0.4186175180, 0.4186175179, 0.4186175182, 0.4186175181, 0.4186175180, 0.4186175179, 0.4186175182]
> Orb(f, x, -10.1, 2000, 2020);
[0.4186175182, 0.4186175181, 0.4186175180, 0.4186175179, 0.4186175182, 0.4186175181, 0.4186175180, 0.4186175179, 0.4186175182, 0.4186175181, 0.4186175180, 0.4186175179,
  0.4186175182, 0.4186175181, 0.4186175180, 0.4186175179, 0.4186175182, 0.4186175181, 0.4186175180, 0.4186175179, 0.4186175182]
> Orb(f, x, 10.1, 2000, 2020);
[0.4186175181, 0.4186175180, 0.4186175179, 0.4186175182, 0.4186175181, 0.4186175180, 0.4186175179, 0.4186175182, 0.4186175181, 0.4186175180, 0.4186175179, 0.4186175182,
  0.4186175181, 0.4186175180, 0.4186175179, 0.4186175182, 0.4186175181, 0.4186175180, 0.4186175179, 0.4186175182, 0.4186175181]
> Orb(f, x, 100.1, 2000, 2020);
[0.4186175182, 0.4186175181, 0.4186175180, 0.4186175179, 0.4186175182, 0.4186175181, 0.4186175180, 0.4186175179, 0.4186175182, 0.4186175181, 0.4186175180, 0.4186175179,
  0.4186175182, 0.4186175181, 0.4186175180, 0.4186175179, 0.4186175182, 0.4186175181, 0.4186175180, 0.4186175179, 0.4186175182]
```

can see the vals converge

3

```
> read ("DMB8.txt")
```

For a list of the Main procedures type: Help8(); for help with a specific procedure type: Help8(ProcedureName); for example Help8(RandomMC);
For a list of the other procedures type: Help8a(); for help with a specific procedure type: Help8(ProcedureName); for example Help8(RandomMC);

```
> Help8( );
```

The available procedures are: Orb, SS, SSS

```
> Help8a( );
```

The other procedures are: MakeNicePol, raRF

```
> f := raRF(x, 3);
```

$$f := \frac{5x^3 + 5x^2 + 3x + 1}{2x^3 + 10x^2 + 10x + 10}$$

```
> SSS(f, x);
  evalf(%);
```

$$[RootOf(2\_Z^4 + 5\_Z^3 + 5\_Z^2 + 7\_Z - 1, index = 1)]$$
$$[0.1292929950]$$

```
> Orb(f, x, -100.1, 2000, 2020);
0.1292929950, 0.1292929950, 0.1292929950, 0.1292929950, 0.1292929950, 0.1292929950, 0.1292929950, 0.1292929950, 0.1292929950, 0.1292929950, 0.1292929950,
  0.1292929950, 0.1292929950, 0.1292929950, 0.1292929950, 0.1292929950, 0.1292929950, 0.1292929950, 0.1292929950, 0.1292929950]
> Orb(f, x, -10.1, 2000, 2020);
0.1292929950, 0.1292929950, 0.1292929950, 0.1292929950, 0.1292929950, 0.1292929950, 0.1292929950, 0.1292929950, 0.1292929950, 0.1292929950, 0.1292929950,
  0.1292929950, 0.1292929950, 0.1292929950, 0.1292929950, 0.1292929950, 0.1292929950, 0.1292929950, 0.1292929950, 0.1292929950]
> Orb(f, x, 10.1, 2000, 2020);
0.1292929950, 0.1292929950, 0.1292929950, 0.1292929950, 0.1292929950, 0.1292929950, 0.1292929950, 0.1292929950, 0.1292929950, 0.1292929950, 0.1292929950,
  0.1292929950, 0.1292929950, 0.1292929950, 0.1292929950, 0.1292929950, 0.1292929950, 0.1292929950, 0.1292929950, 0.1292929950]
> Orb(f, x, 100.1, 2000, 2020);
0.1292929950, 0.1292929950, 0.1292929950, 0.1292929950, 0.1292929950, 0.1292929950, 0.1292929950, 0.1292929950, 0.1292929950, 0.1292929950, 0.1292929950,
  0.1292929950, 0.1292929950, 0.1292929950, 0.1292929950, 0.1292929950, 0.1292929950, 0.1292929950, 0.1292929950, 0.1292929950]
```

can see the values conv

2. Find the terms between 2000-th and 2030-th terms of the **orbit** starting at $x_0 = 0.5$ of the recurrence

$$x_n = kx_{n-1}(1 - x_{n-1}) \quad ,$$

for $k = 1$, $k = 2$, $k = 2.5$, $k = 3.1$, $k = 3.2$, $k = 3.3$, $k = 3.5$. What do you find?

converge

```
> f := x·(1 − x);
```

$$f := x\,(1 - x) \tag{10}$$

```
> Orb(f, x, .5, 2000, 2030);
```

[0.0004976675995, 0.0004974199265, 0.0004971724999, 0.0004969253194, 0.0004966783846, 0.0004964316952, 0.0004961852508, 0.0004959390510, 0.0004956930954, 0.0004954473838, 0.0004952019157, 0.0004949566908, 0.0004947117087, 0.0004944669690, 0.0004942224714, 0.0004939782155, 0.0004937342010, 0.0004934904275, 0.0004932468947, 0.0004930036022, 0.0004927605496, 0.0004925177367, 0.0004922751630, 0.0004920328281, 0.0004917907318, 0.0004915488737, 0.0004913072534, 0.0004910658706, 0.0004908247249, 0.0004905838160, 0.0004903431435] (11)

```
> f := 2·x·(1 − x);
```

$$f := 2\,x\,(1 - x) \tag{12}$$

```
> Orb(f, x, .5, 2000, 2030);
```

[0.5000000000, 0.5000000000] (13)

```
> f := 2.5·x·(1 − x);
```

$$f := 2.5\,x\,(1 - x) \tag{14}$$

```
> Orb(f, x, .5, 2000, 2030);
```

[0.6000000000, 0.6000000000] (15)

```
> f := 3.1·x·(1 − x);
```

$$f := 3.1\,x\,(1 - x) \tag{16}$$

```
> Orb(f, x, .5, 2000, 2030);
```

[0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245] (17)

```
> f := 3.2·x·(1 − x);
```

$$f := 3.2\,x\,(1 - x) \tag{18}$$

```
> Orb(f, x, .5, 2000, 2030);
```

[0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091] (19)

```
> f := 3.3·x·(1 − x);
```

$$f := 3.3\,x\,(1 - x) \tag{20}$$

```
> Orb(f, x, .5, 2000, 2030);
```

[0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198] (21)

```
> f := 3.5·x·(1 − x);
```

$$f := 3.5\,x\,(1 - x) \tag{22}$$

```
> Orb(f, x, .5, 2000, 2030);
```

[0.5008842111, 0.8749972637, 0.3828196827, 0.8269407062, 0.5008842111, 0.8749972637, 0.3828196827, 0.8269407062, 0.5008842111, 0.8749972637, 0.3828196827, 0.8269407062, 0.5008842111, 0.8749972637, 0.3828196827, 0.8269407062, 0.5008842111, 0.8749972637, 0.3828196827, 0.8269407062, 0.5008842111, 0.8749972637, 0.3828196827, 0.8269407062, 0.5008842111, 0.8749972637, 0.3828196827, 0.8269407062, 0.5008842111, 0.8749972637, (23)

lesser k values such as 1, 2, and 2.5 converge to a stable steady state.

As k increases, we observe periodic behavior.

3. By Playing "High-Low" find the smallest positive integer such that `k`,

```
f:=MakeNicePol([1,2,3,4,5,6,7,8],k,x);
```

has two stable steady-states. Pick random `x0` from their neighborhood and verify that indeed the orbits converges to them.

Then keep upping k to find the smallest integer such that it has four stable steady-states. Pick random `x0` from their neighborhood and verify that indeed the orbits converges to them.

361

```
read "DMB8.txt" :
f := MakeNicePol([1, 2, 3, 4, 5, 6, 7, 8], 361, x);
f;
```

For a list of the Main procedures type: Help8(); for help with a specific procedure type: Help8(ProcedureName); for example Help8(RandomMC);
For a list of the other procedures type: Help8a(); for help with a specific procedure type: Help8(ProcedureName); for example Help8(RandomMC);

$$f := \frac{1}{361}x^9 - \frac{36}{361}x^8 + \frac{546}{361}x^7 - \frac{4536}{361}x^6 + \frac{22449}{361}x^5 - \frac{67284}{361}x^4 + \frac{118124}{361}x^3 - \frac{109584}{361}x^2 + \frac{40681}{361}x$$

$$\frac{1}{361}x^9 - \frac{36}{361}x^8 + \frac{546}{361}x^7 - \frac{4536}{361}x^6 + \frac{22449}{361}x^5 - \frac{67284}{361}x^4 + \frac{118124}{361}x^3 - \frac{109584}{361}x^2 + \frac{40681}{361}x$$

```
SSS(f, x);
```

[3, 5]

```
Orb(f, x, 3.5, 1000, 1010)
```

[3.0003042, 2.9997023, 3.0003023, 2.9996972, 3.0003126, 2.9996899, 3.0003070, 2.9996928, 3.0003148, 2.9996848, 3.0003143]

```
Orb(f, x, 5.1, 1000, 1010)
```

[5.0005525, 4.9994615, 5.0005980, 4.9994259, 5.0006803, 4.9993623, 5.0005312, 4.9993682, 5.0006491, 4.9992884, 5.0006114]

2521

```
read "DMB8.txt" :
f := MakeNicePol([1, 2, 3, 4, 5, 6, 7, 8], 2521, x);
f;
```

For a list of the Main procedures type: Help8(); for help with a specific procedure type: Help8(ProcedureName); for example Help8(RandomMC);
For a list of the other procedures type: Help8a(); for help with a specific procedure type: Help8(ProcedureName); for example Help8(RandomMC);

$$f := \frac{1}{2521}x^9 - \frac{36}{2521}x^8 + \frac{546}{2521}x^7 - \frac{4536}{2521}x^6 + \frac{22449}{2521}x^5 - \frac{67284}{2521}x^4 + \frac{118124}{2521}x^3 - \frac{109584}{2521}x^2 + \frac{42841}{2521}x$$

$$\frac{1}{2521}x^9 - \frac{36}{2521}x^8 + \frac{546}{2521}x^7 - \frac{4536}{2521}x^6 + \frac{22449}{2521}x^5 - \frac{67284}{2521}x^4 + \frac{118124}{2521}x^3 - \frac{109584}{2521}x^2 + \frac{42841}{2521}x$$

```
SSS(f, x);
```

[1, 3, 5, 7]

```
> SSS(f, x)
```

[1, 3, 5, 7]

```
> Orb(f, x, 1.1, 1000, 1010)
```

[1.00446666, 0.99560040, 1.00445773, 0.99560910, 1.00444881, 0.99561775, 1.00443992, 0.99562636, 1.00443108, 0.99563496, 1.00442225]

```
> Orb(f, x, 3.1, 1000, 1010)
```

[2.99999944, 2.99999942, 2.99999928, 2.99999940, 2.99999914, 2.99999952, 2.99999988, 3.00000010, 3.00000054, 2.99999921, 2.99999851]

```
> Orb(f, x, 5.1, 1000, 1010)
```

[4.99998819, 5.00000686, 4.99999614, 4.99999196, 5.00000793, 4.99999232, 4.99998305, 4.99999152, 4.99998645, 4.99997930, 4.99997679]

```
> Orb(f, x, 7.1, 1000, 1010)
```

[7.0092564, 6.9905882, 7.0092033, 6.9905529, 7.0092564, 6.9905882, 7.0092033, 6.9905529, 7.0092564, 6.9905882, 7.0092033]