HOMEWORK 8 DMB: EZRA CHECHIK

1. Ten random functions using raRF(x,3), and stable steady states and orbits:

$$\left[\left[\frac{9x^3+2x^2+4x+8}{4x^3+7x^2+9x+5}, [0.9127501374], [0.9127501374, 0.9127501374, 0.9127501374, 0.9127501369]\right], \left[\frac{10x^3+9x^2+10x+3}{8x^3+7x^2+9x+4}, [1.164908077], [1.164908076, 1.164908076, 1.164908076, 1.164908076]\right], \left[\frac{2x^3+10x^2+x+9}{x^3+8x^2+7x+4}, [1.094863281], [1.094863281, 1.094863281, 1.094863281, 1.094863281]\right], \left[\frac{7x^3+x^2+4x+6}{9x^3+5x^2+9x+2}, [0.7927386281], [0.7927386279, 0.7927386279, 0.7927386279, 0.7927386279]\right], \left[\frac{4x^3+3x^2+4x+9}{10x^3+4x^2+4x+6}, [0.8947709009], [0.8947709013, 0.8947709013, 0.8947709013, 0.8947709013]\right], \left[\frac{2x^3+7x^2+2x+8}{7x^3+5x^2+3x+5}, [0.9672041816], [0.9672041817, 0.9672041817, 0.9672041817, 0.9672041817]\right], \left[\frac{8x^3+6x^2+4x+5}{8x^3+6x^2+2x+10}, [0.8389158560], [0.8389158560, 0.8389158560, 0.8389158560, 0.8389158560]\right], \left[\frac{9x^3+7x^2+8x+4}{2x^3+3x^2+10x+4}, [2.809650662], [2.809650663, 2.809650663, 2.809650663, 2.809650663]\right], \left[\frac{5x^3+x^2+x+9}{x^3+4x^2+4x+2}, [1.441649193], [1.441649193, 1.441649193, 1.441649193, 1.441649193]\right], \left[\frac{7x^3+x^2+3x+6}{10x^3+5x^2+x+2}, [0.9680653581], [0.9680653584, 0.9680653589, 0.9680653584, 0.9680653584]\right]\right]$$

Analysis of orbits: *Function number, 1:* all orbits converge to **~ 0.9127;** *Function number, 2:* all orbits converge to **~ 1.1649**; *Function number, 3:* all orbits converge to **~ 1.0948**; *Function number, 4:* all orbits converge to **~ 0.7927**; *Function number, 5:* all orbits converge to **~ 0.8947**; *Function number, 6:* all orbits converge to **~ 0.9672**; *Function number, 7:* all orbits converge to **~ 0.8389**; *Function number, 8:* all orbits converge to **~ 2.8096**; *Function number, 9:* all orbits converge to **~ 1.4416**; *Function number, 10:* all orbits converge to **~ 0.9680.**

2. Findings: K = 1: converges to 0; K = 2: coverges to 0.5, K = 2.5: converges to 0.6; K = 3.1; alternates between two values; K = 3.2: alternates between two values; K = 3.3: alternates between two values; K = 3.5: alternates between four values.

$k =, 1$

Orbit terms (n = 2000..2030):, [0.0004976675995, 0.0004974199265, 0.0004971724999, 0.0004969253194, 0.0004966783846, 0.0004964316952, 0.0004961852508, 0.0004959390510, 0.0004956930954, 0.0004954473838, 0.0004952019157, 0.0004949566908, 0.0004947117087, 0.0004944669690, 0.0004942224714, 0.0004939782155, 0.0004937342010, 0.0004934904275, 0.0004932468947, 0.0004930036022, 0.0004927605496, 0.0004925177367, 0.0004922751630, 0.0004920328281, 0.0004917907318, 0.0004915488737, 0.0004913072534, 0.0004910658706, 0.0004908247249, 0.0004905838160, 0.0004903431435]

$k =, 2$

Orbit terms (n = 2000..2030):, [0.5000000000, 0.5000000000]

==========

$k =, 2.5$

Orbit terms (n = 2000..2030):, [0.6000000000, 0.6000000000]

==========

$k =, 3.1$

Orbit terms (n = 2000..2030):, [0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245]

==========

$k =, 3.2$

Orbit terms (n = 2000..2030):, [0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091]

==========

$k =, 3.3$

Orbit terms (n = 2000..2030):, [0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198]

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$k =, 3.5$

Orbit terms (n = 2000..2030):, [0.5008842111, 0.8749972637, 0.3828196827, 0.8269407062, 0.5008842111, 0.8749972637, 0.3828196827, 0.8269407062, 0.5008842111, 0.8749972637, 0.3828196827, 0.8269407062, 0.5008842111, 0.8749972637, 0.3828196827, 0.8269407062, 0.5008842111, 0.8749972637, 0.3828196827, 0.8269407062, 0.5008842111, 0.8749972637, 0.3828196827, 0.8269407062, 0.5008842111, 0.8749972637, 0.3828196827, 0.8269407062, 0.5008842111, 0.8749972637, 0.3828196827]

3. Smallest positive int. K such that we get 2 SSS:

```
> k := 500 :
f := MakeNicePol([1, 2, 3, 4, 5, 6, 7, 8], k, x) :
while nops(SSS(f, x)) ≥ 2 do
   k := k − 1;
  f := MakeNicePol(L, k, x);
od:

 # Last k that had exactly 2 SSS is k+1
k_last := k + 1 :
print(`Smallest k with exactly 2 stable steady−states:`, k_last);
```

Smallest k with exactly 2 stable steady-states:, 361 **(2)**

```
> evalf(Orb(f_last, x, 2.99, 2000, 2050));
```

[2.9996865, 3.0003038, 2.9996942, 3.0003095, 2.9996875, 3.0003135, 2.9996843, 3.0003129, 2.9996797, 3.0003235, 2.9996722, 3.0003294, 2.9996711, 3.0003244, 2.9996776, 3.0003249, 2.9996760, 3.0003166, 2.9996836, 3.0003200, 2.9996888, 3.0003030, 2.9996971, 3.0003073, 2.9996926, 3.0003042, 2.9997023, 3.0003023, 2.9996972, 3.0003126, 2.9996899, 3.0003070, 2.9996928, 3.0003148, 2.9996848, 3.0003143, 2.9996865, 3.0003038, 2.9996942, 3.0003095, 2.9996875, 3.0003135, 2.9996843, 3.0003129, 2.9996797, 3.0003235, 2.9996722, 3.0003294, 2.9996711, 3.0003244, 2.9996776] **(5)**

```
> evalf(Orb(f_last, x, 3.01, 2000, 2050));
```

[3.0003095, 2.9996875, 3.0003135, 2.9996843, 3.0003129, 2.9996797, 3.0003235, 2.9996722, 3.0003294, 2.9996711, 3.0003244, 2.9996776, 3.0003249, 2.9996760, 3.0003166, 2.9996836, 3.0003200, 2.9996888, 3.0003030, 2.9996971, 3.0003073, 2.9996926, 3.0003042, 2.9997023, 3.0003023, 2.9996972, 3.0003126, 2.9996899, 3.0003070, 2.9996928, 3.0003148, 2.9996848, 3.0003143, 2.9996865, 3.0003038, 2.9996942, 3.0003095, 2.9996875, 3.0003135, 2.9996843, 3.0003129, 2.9996797, 3.0003235, 2.9996722, 3.0003294, 2.9996711, 3.0003244, 2.9996776, 3.0003249, 2.9996760, 3.0003166] **(6)**

```
> evalf(Orb(f_last, x, 4.99, 2000, 2050));
```

[4.9994259, 5.0006803, 4.9993623, 5.0005312, 4.9993682, 5.0006491, 4.9992884, 5.0006114, 4.9992760, 5.0008421, 4.9992045, 5.0006438, 4.9992821, 5.0005525, 4.9994615, 5.0005980, 4.9994259, 5.0006803, 4.9993623, 5.0005312, 4.9993682, 5.0006491, 4.9992884, 5.0006114, 4.9992760, 5.0008421, 4.9992045, 5.0006438, 4.9992821, 5.0005525, 4.9994615, 5.0005980, 4.9994259, 5.0006803, 4.9993623, 5.0005312, 4.9993682, 5.0006491, 4.9992884, 5.0006114, 4.9992760, 5.0008421, 4.9992045, 5.0006438, 4.9992821, 5.0005525, 4.9994615, 5.0005980, 4.9994259, 5.0006803, 4.9993623] **(7)**

```
> evalf(Orb(f_last, x, 5.01, 2000, 2050));
```

[5.0005525, 4.9994615, 5.0005980, 4.9994259, 5.0006803, 4.9993623, 5.0005312, 4.9993682, 5.0006491, 4.9992884, 5.0006114, 4.9992760, 5.0008421, 4.9992045, 5.0006438, 4.9992821, 5.0005525, 4.9994615, 5.0005980, 4.9994259, 5.0006803, 4.9993623, 5.0005312, 4.9993682, 5.0006491, 4.9992884, 5.0006114, 4.9992760, 5.0008421, 4.9992045, 5.0006438, 4.9992821, 5.0005525, 4.9994615, 5.0005980, 4.9994259, 5.0006803, 4.9993623, 5.0005312, 4.9993682, 5.0006491, 4.9992884, 5.0006114, 4.9992760, 5.0008421, 4.9992045, 5.0006438, 4.9992821, 5.0005525, 4.9994615, 5.0005980] **(8)**

Smallest positive int. K such that we get 4 SSS

```
> k := 1 :
L := [1, 2, 3, 4, 5, 6, 7, 8] :  f := MakeNicePol(L, k, x) :
while nops(SSS(f, x)) ≠ 4 do
   k := k + 1;
  f := MakeNicePol(L, k, x);
od:

 # Last k that had exactly 2 SSS is k
print(`Smallest k with exactly 4 stable steady−states:`, k);
```

Smallest k with exactly 4 stable steady-states:, 2521 **(2)**

```
> f_last := MakeNicePol(L, k, x);
```

$$f\_last := \frac{1}{2521}x^9 - \frac{36}{2521}x^8 + \frac{546}{2521}x^7 - \frac{4536}{2521}x^6 + \frac{22449}{2521}x^5 - \frac{67284}{2521}x^4 + \frac{118124}{2521}x^3 - \frac{109584}{2521}x^2 + \frac{42841}{2521}x \quad \textbf{(3)}$$

```
> StablePts := evalf(SSS(f_last, x))
```

StablePts := [1., 3., 5., 7.] **(4)**

```
> evalf(Orb(f_last, x, 0.99, 2000, 2050));
[0.99862280, 1.00138214, 0.99862509, 1.00137984, 0.99862729, 1.00137762, 0.99862952, 1.00137538, 0.99863175, 1.00137313, 0.99863400, 1.00137086, 0.99863621, 1.00136862, 0.99863845, 1.00136637, 0.99864063,
    1.00136417, 0.99864281, 1.00136198, 0.99864503, 1.00135973, 0.99864723, 1.00135753, 0.99864942, 1.00135531, 0.99865160, 1.00135312, 0.99865375, 1.00135095, 0.99865596, 1.00134872, 0.99865817, 1.00134649,
    0.99866038, 1.00134427, 0.99866257, 1.00134207, 0.99866473, 1.00133989, 0.99866691, 1.00133769, 0.99866911, 1.00133547, 0.99867132, 1.00133325, 0.99867344, 1.00133110, 0.99867560, 1.00132894, 0.99867776]   (5)
> evalf(Orb(f_last, x, 1.01, 2000, 2050));
[1.00136406, 0.99864296, 1.00136181, 0.99864517, 1.00135959, 0.99864738, 1.00135738, 0.99864959, 1.00135515, 0.99865179, 1.00135292, 0.99865402, 1.00135068, 0.99865619, 1.00134850, 0.99865838, 1.00134629,
    0.99866057, 1.00134408, 0.99866272, 1.00134193, 0.99866488, 1.00133974, 0.99866702, 1.00133758, 0.99866920, 1.00133539, 0.99867135, 1.00133322, 0.99867353, 1.00133102, 0.99867566, 1.00132888, 0.99867779,
    1.00132672, 0.99867997, 1.00132453, 0.99868209, 1.00132239, 0.99868418, 1.00132028, 0.99868634, 1.00131810, 0.99868849, 1.00131595, 0.99869058, 1.00131383, 0.99869272, 1.00131168, 0.99869488, 1.00130952]   (6)
> evalf(Orb(f_last, x, 2.99, 2000, 2050));
[2.99999926, 2.99999926, 2.99999926, 2.99999926, 2.99999926, 2.99999926, 2.99999926, 2.99999926, 2.99999926, 2.99999926, 2.99999926, 2.99999926, 2.99999926, 2.99999926, 2.99999926, 2.99999926, 2.99999926,
    2.99999926, 2.99999926, 2.99999926, 2.99999926, 2.99999926, 2.99999926, 2.99999926, 2.99999926, 2.99999926, 2.99999926, 2.99999926, 2.99999926, 2.99999926, 2.99999926, 2.99999926, 2.99999926, 2.99999926,
    2.99999926, 2.99999926, 2.99999926, 2.99999926, 2.99999926, 2.99999926, 2.99999926, 2.99999926, 2.99999926, 2.99999926, 2.99999926, 2.99999926, 2.99999926, 2.99999926, 2.99999926, 2.99999926, 2.99999926]   (7)
> evalf(Orb(f_last, x, 3.01, 2000, 2050));
[3.00000230, 3.00000212, 3.00000196, 3.00000204, 3.00000150, 3.00000213, 3.00000153, 3.00000184, 3.00000230, 3.00000212, 3.00000196, 3.00000204, 3.00000150, 3.00000213, 3.00000153, 3.00000184, 3.00000230,
    3.00000212, 3.00000196, 3.00000204, 3.00000150, 3.00000213, 3.00000153, 3.00000184, 3.00000230, 3.00000212, 3.00000196, 3.00000204, 3.00000150, 3.00000213, 3.00000153, 3.00000184, 3.00000230, 3.00000212,
    3.00000196, 3.00000204, 3.00000150, 3.00000213, 3.00000153, 3.00000184, 3.00000230, 3.00000212, 3.00000196, 3.00000204, 3.00000150, 3.00000213, 3.00000153, 3.00000184, 3.00000230, 3.00000212, 3.00000196]   (8)
> evalf(Orb(f_last, x, 4.99, 2000, 2050));
[5.00000584, 4.99999780, 5.00001217, 5.00001137, 5.00000378, 4.99998980, 4.99998922, 4.99998437, 4.99998795, 5.00000379, 5.00000697, 4.99998901, 4.99999580, 4.99998819, 5.00000686, 4.99999614, 4.99999196,
    5.00000793, 4.99999232, 4.99998305, 4.99999152, 4.99998645, 4.99997930, 4.99997679, 4.99996714, 4.99997215, 4.99997929, 4.99999762, 4.99999412, 4.99998664, 4.99999452, 4.99999543, 5.00001090, 5.00000679,
    5.00001695, 5.00000260, 4.99999474, 4.99998617, 4.99998154, 4.99999286, 4.99999523, 5.00000150, 5.00000905, 5.00001035, 5.00002544, 5.00000888, 5.00002146, 5.00000524, 4.99999961, 4.99998893, 4.99998844]   (9)
> evalf(Orb(f_last, x, 5.01, 2000, 2050));
[4.99999474, 4.99998617, 4.99998154, 4.99999286, 4.99999523, 5.00000150, 5.00000905, 5.00001035, 5.00002544, 5.00000888, 5.00002146, 5.00000524, 4.99999961, 4.99998893, 4.99998844, 4.99997911, 4.99997056,
    4.99997327, 4.99998732, 5.00000008, 5.00000192, 4.99999919, 5.00000380, 5.00001114, 4.99999887, 5.00000336, 4.99999866, 4.99999479, 5.00000902, 5.00000484, 5.00000181, 4.99999132, 5.00000706, 5.00000554,
    5.00000870, 5.00000440, 5.00000833, 5.00001112, 5.00000753, 5.00001252, 5.00002232, 5.00003786, 5.00001794, 5.00000543, 5.00000584, 4.99999780, 5.00001217, 5.00001137, 5.00000378, 4.99998980, 4.99998922]   (10)
> evalf(Orb(f_last, x, 6.99, 2000, 2050));
[6.9969269, 7.0030610, 6.9968658, 7.0030687, 6.9968786, 7.0031102, 6.9968778, 7.0031236, 6.9968676, 7.0029743, 6.9971874, 7.0029548, 6.9969580, 7.0031515, 6.9967947, 7.0032294, 6.9968105, 7.0030469, 6.9970111,
    7.0028829, 6.9969802, 7.0030347, 6.9969508, 7.0030781, 6.9969463, 7.0030297, 6.9969699, 7.0032447, 6.9967955, 7.0032160, 6.9967018, 7.0034297, 6.9965723, 7.0034690, 6.9963622, 7.0035927, 6.9964573, 7.0034248,
    6.9965820, 7.0033939, 6.9965909, 7.0034621, 6.9964259, 7.0033802, 6.9966191, 7.0033083, 6.9967833, 7.0029797, 6.9970412, 7.0028354, 6.9971310]   (11)
> evalf(Orb(f_last, x, 7.01, 2000, 2050));
[7.0031486, 6.9967414, 7.0032847, 6.9967512, 7.0031702, 6.9967675, 7.0033622, 6.9964432, 7.0034712, 6.9965496, 7.0034013, 6.9965507, 7.0035010, 6.9965220, 7.0033503, 6.9964260, 7.0034609, 6.9966755, 7.0033008,
    6.9967218, 7.0031136, 6.9968256, 7.0031485, 6.9967907, 7.0031345, 6.9969158, 7.0029544, 6.9970942, 7.0029560, 6.9970074, 7.0029210, 6.9970016, 7.0028404, 6.9971320, 7.0027834, 6.9971653, 7.0030023, 6.9968362,
    7.0030207, 6.9969269, 7.0030610, 6.9968658, 7.0030687, 6.9968786, 7.0031102, 6.9968778, 7.0031236, 6.9968676, 7.0029743, 6.9971874, 7.0029548]   (12)
>
```

* Orbits show attraction → all are confirmed SSS