

> $funcs := map(_ \rightarrow raRF(x, 3), [0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10])$

$$funcs := \left[\frac{3x^3 + 3x^2 + 7x + 7}{10x^3 + 8x^2 + 3x + 5}, \frac{3x^3 + x^2 + x + 3}{10x^3 + 10x^2 + 5x + 5}, \frac{4x^3 + 8x^2 + 2x + 10}{9x^3 + 5x^2 + 9x + 2}, \right. \\ \left. \frac{10x^3 + 3x^2 + 4x + 7}{9x^3 + 4x^2 + 10x + 9}, \frac{x^3 + 9x^2 + 8x + 7}{7x^3 + 4x^2 + 2x + 10}, \frac{4x^3 + 6x^2 + x + 8}{9x^3 + 2x^2 + 7x + 1}, \right. \\ \left. \frac{4x^3 + 9x^2 + 9x + 5}{4x^3 + 6x^2 + 4x + 3}, \frac{2x^3 + 8x^2 + 10x + 4}{3x^3 + 5x^2 + 2x + 7}, \frac{4x^3 + 5x^2 + 7x + 5}{2x^3 + 10x^2 + 8x + 6}, \right. \\ \left. \frac{8x^3 + 4x^2 + 8x + 6}{10x^3 + 4x^2 + 9x + 7}, \frac{x^3 + 9x^2 + 2x + 3}{4x^3 + 2x^2 + 5x + 1} \right] \quad (1)$$

> $stableSteadyStates = map(f \rightarrow evalf(SSS(f, x)), funcs)$

$$stableSteadyStates = [[0.8620096788], [0.4042380413], [0.9747201978], [0.6982012560], \\ [1.070745313], [1.], [1.460448809], [1.406545762], [0.7994227258], [0.8696205581], \\ [1.197066008, -1.260488554]] \quad (2)$$

> **for** x_0 **in** $[-100.1, -10.1, 10.1, 100.1]$ **do**

for f **in** $funcs$ **do**

$print('x0 = ', x_0); print('f = ', f) :$

$print('SSS := ', evalf(SSS(f, x))) :$

$print('Orb = ', Orb(f, x, x_0, 2000, 2020)) :$

end

end:

$$x0 = , -100.1$$

$$f = , \frac{3x^3 + 3x^2 + 7x + 7}{10x^3 + 8x^2 + 3x + 5}$$

$$SSS := , [0.8620096788]$$

$$Orb = , [0.8620096794, 0.8620096783, 0.8620096794, 0.8620096783, 0.8620096794, \\ 0.8620096783, 0.8620096794, 0.8620096783, 0.8620096794, 0.8620096783, 0.8620096794, \\ 0.8620096783, 0.8620096794, 0.8620096783, 0.8620096794, 0.8620096783, 0.8620096794, \\ 0.8620096783, 0.8620096794, 0.8620096783, 0.8620096794]$$

$$x0 = , -100.1$$

$$f = , \frac{3x^3 + x^2 + x + 3}{10x^3 + 10x^2 + 5x + 5}$$

$$SSS := , [0.4042380413]$$

$$Orb = , [0.4042380416, 0.4042380414, 0.4042380416, 0.4042380414, 0.4042380416, \\ 0.4042380414, 0.4042380416, 0.4042380414, 0.4042380416, 0.4042380414, 0.4042380416, \\ 0.4042380414, 0.4042380416, 0.4042380414, 0.4042380416, 0.4042380414, 0.4042380416, \\ 0.4042380414, 0.4042380416, 0.4042380414, 0.4042380416]$$

0.4042380414, 0.4042380416, 0.4042380414, 0.4042380416, 0.4042380414, 0.4042380416,
0.4042380414, 0.4042380416, 0.4042380414, 0.4042380416, 0.4042380414, 0.4042380416,
0.4042380414, 0.4042380416, 0.4042380414, 0.4042380416]

$$x0 = , -100.1$$

$$f = , \frac{4x^3 + 8x^2 + 2x + 10}{9x^3 + 5x^2 + 9x + 2}$$

$$SSS := , [0.9747201978]$$

Orb = , [0.9747201982, 0.9747201978, 0.9747201982, 0.9747201978, 0.9747201982,
0.9747201978, 0.9747201982, 0.9747201978, 0.9747201982, 0.9747201978, 0.9747201982,
0.9747201978, 0.9747201982, 0.9747201978, 0.9747201982, 0.9747201978, 0.9747201982,
0.9747201978, 0.9747201982, 0.9747201978, 0.9747201982]

$$x0 = , -100.1$$

$$f = , \frac{10x^3 + 3x^2 + 4x + 7}{9x^3 + 4x^2 + 10x + 9}$$

$$SSS := , [0.6982012560]$$

Orb = , [0.6982012559, 0.6982012561, 0.6982012562, 0.6982012559, 0.6982012561,
0.6982012562, 0.6982012559, 0.6982012561, 0.6982012562, 0.6982012559, 0.6982012561,
0.6982012562, 0.6982012559, 0.6982012561, 0.6982012562, 0.6982012559, 0.6982012561,
0.6982012562, 0.6982012559, 0.6982012561, 0.6982012562]

$$x0 = , -100.1$$

$$f = , \frac{x^3 + 9x^2 + 8x + 7}{7x^3 + 4x^2 + 2x + 10}$$

$$SSS := , [1.070745313]$$

Orb = , [1.070745313, 1.070745313, 1.070745313, 1.070745313, 1.070745313, 1.070745313,
1.070745313, 1.070745313, 1.070745313, 1.070745313, 1.070745313, 1.070745313,
1.070745313, 1.070745313, 1.070745313, 1.070745313, 1.070745313, 1.070745313,
1.070745313, 1.070745313, 1.070745313]

$$x0 = , -100.1$$

$$f = , \frac{4x^3 + 6x^2 + x + 8}{9x^3 + 2x^2 + 7x + 1}$$

$$SSS := , [1.]$$

Orb = , [1.000000001, 0.9999999989, 1.000000001, 0.9999999989, 1.000000001, 0.9999999989,
1.000000001, 0.9999999989, 1.000000001, 0.9999999989, 1.000000001, 0.9999999989,

1.0000000001, 0.9999999989, 1.0000000001, 0.9999999989, 1.0000000001, 0.9999999989,
1.0000000001, 0.9999999989, 1.0000000001]

$$x0 = , -100.1$$

$$f = , \frac{4x^3 + 9x^2 + 9x + 5}{4x^3 + 6x^2 + 4x + 3}$$

$$SSS := , [1.460448809]$$

Orb = , [1.460448809, 1.460448809, 1.460448809, 1.460448809, 1.460448809, 1.460448809,
1.460448809, 1.460448809, 1.460448809, 1.460448809, 1.460448809, 1.460448809,
1.460448809, 1.460448809, 1.460448809, 1.460448809, 1.460448809, 1.460448809,
1.460448809, 1.460448809, 1.460448809]

$$x0 = , -100.1$$

$$f = , \frac{2x^3 + 8x^2 + 10x + 4}{3x^3 + 5x^2 + 2x + 7}$$

$$SSS := , [1.406545762]$$

Orb = , [1.406545762, 1.406545762, 1.406545762, 1.406545762, 1.406545762, 1.406545762,
1.406545762, 1.406545762, 1.406545762, 1.406545762, 1.406545762, 1.406545762,
1.406545762, 1.406545762, 1.406545762, 1.406545762, 1.406545762, 1.406545762,
1.406545762, 1.406545762, 1.406545762]

$$x0 = , -100.1$$

$$f = , \frac{4x^3 + 5x^2 + 7x + 5}{2x^3 + 10x^2 + 8x + 6}$$

$$SSS := , [0.7994227258]$$

Orb = , [0.7994227260, 0.7994227260, 0.7994227260, 0.7994227260, 0.7994227260,
0.7994227260, 0.7994227260, 0.7994227260, 0.7994227260, 0.7994227260, 0.7994227260,
0.7994227260, 0.7994227260, 0.7994227260, 0.7994227260, 0.7994227260, 0.7994227260,
0.7994227260, 0.7994227260, 0.7994227260]

$$x0 = , -100.1$$

$$f = , \frac{8x^3 + 4x^2 + 8x + 6}{10x^3 + 4x^2 + 9x + 7}$$

$$SSS := , [0.8696205581]$$

Orb = , [0.8696205577, 0.8696205577, 0.8696205577, 0.8696205577, 0.8696205577,
0.8696205577, 0.8696205577, 0.8696205577, 0.8696205577, 0.8696205577, 0.8696205577,
0.8696205577, 0.8696205577, 0.8696205577, 0.8696205577, 0.8696205577, 0.8696205577,
0.8696205577, 0.8696205577, 0.8696205577]

0.8696205577, 0.8696205577, 0.8696205577, 0.8696205577]

$$x0 = , -100.1$$

$$f = , \frac{x^3 + 9x^2 + 2x + 3}{4x^3 + 2x^2 + 5x + 1}$$

$$SSS := , [1.197066008, -1.260488554]$$

Orb = , [1.197066008, 1.197066007, 1.197066008, 1.197066007, 1.197066008, 1.197066007,
1.197066008, 1.197066007, 1.197066008, 1.197066007, 1.197066008, 1.197066007,
1.197066008, 1.197066007, 1.197066008, 1.197066007, 1.197066008, 1.197066007,
1.197066008, 1.197066007, 1.197066008]

$$x0 = , -10.1$$

$$f = , \frac{3x^3 + 3x^2 + 7x + 7}{10x^3 + 8x^2 + 3x + 5}$$

$$SSS := , [0.8620096788]$$

Orb = , [0.8620096794, 0.8620096783, 0.8620096794, 0.8620096783, 0.8620096794,
0.8620096783, 0.8620096794, 0.8620096783, 0.8620096794, 0.8620096783, 0.8620096794,
0.8620096783, 0.8620096794, 0.8620096783, 0.8620096794, 0.8620096783, 0.8620096794,
0.8620096783, 0.8620096794, 0.8620096783, 0.8620096794]

$$x0 = , -10.1$$

$$f = , \frac{3x^3 + x^2 + x + 3}{10x^3 + 10x^2 + 5x + 5}$$

$$SSS := , [0.4042380413]$$

Orb = , [0.4042380416, 0.4042380414, 0.4042380416, 0.4042380414, 0.4042380416,
0.4042380414, 0.4042380416, 0.4042380414, 0.4042380416, 0.4042380414, 0.4042380416,
0.4042380414, 0.4042380416, 0.4042380414, 0.4042380416, 0.4042380414, 0.4042380416,
0.4042380414, 0.4042380416, 0.4042380414, 0.4042380416]

$$x0 = , -10.1$$

$$f = , \frac{4x^3 + 8x^2 + 2x + 10}{9x^3 + 5x^2 + 9x + 2}$$

$$SSS := , [0.9747201978]$$

Orb = , [0.9747201986, 0.9747201970, 0.9747201986, 0.9747201970, 0.9747201986,
0.9747201970, 0.9747201986, 0.9747201970, 0.9747201986, 0.9747201970, 0.9747201986,
0.9747201970, 0.9747201986, 0.9747201970, 0.9747201986, 0.9747201970, 0.9747201986,
0.9747201970, 0.9747201986, 0.9747201970, 0.9747201986]

$$x0 = , -10.1$$

$$f = , \frac{10 x^3 + 3 x^2 + 4 x + 7}{9 x^3 + 4 x^2 + 10 x + 9}$$

$$SSS := , [0.6982012560]$$

$$Orb = , [0.6982012559, 0.6982012561, 0.6982012562, 0.6982012559, 0.6982012561, 0.6982012562, 0.6982012559, 0.6982012561, 0.6982012562, 0.6982012559, 0.6982012561, 0.6982012562, 0.6982012559, 0.6982012561, 0.6982012562]$$

$$x0 = , -10.1$$

$$f = , \frac{x^3 + 9 x^2 + 8 x + 7}{7 x^3 + 4 x^2 + 2 x + 10}$$

$$SSS := , [1.070745313]$$

$$Orb = , [1.070745313, 1.070745313, 1.070745313, 1.070745313, 1.070745313, 1.070745313, 1.070745313, 1.070745313, 1.070745313, 1.070745313, 1.070745313, 1.070745313, 1.070745313, 1.070745313, 1.070745313]$$

$$x0 = , -10.1$$

$$f = , \frac{4 x^3 + 6 x^2 + x + 8}{9 x^3 + 2 x^2 + 7 x + 1}$$

$$SSS := , [1.]$$

$$Orb = , [1.000000001, 0.9999999989, 1.000000001, 0.9999999989, 1.000000001, 0.9999999989, 1.000000001, 0.9999999989, 1.000000001, 0.9999999989, 1.000000001, 0.9999999989, 1.000000001, 0.9999999989, 1.000000001]$$

$$x0 = , -10.1$$

$$f = , \frac{4 x^3 + 9 x^2 + 9 x + 5}{4 x^3 + 6 x^2 + 4 x + 3}$$

$$SSS := , [1.460448809]$$

$$Orb = , [1.460448809, 1.460448809, 1.460448809, 1.460448809, 1.460448809, 1.460448809, 1.460448809, 1.460448809, 1.460448809, 1.460448809, 1.460448809, 1.460448809, 1.460448809, 1.460448809, 1.460448809]$$

$$x0 = , -10.1$$

$$f =, \frac{2 x^3 + 8 x^2 + 10 x + 4}{3 x^3 + 5 x^2 + 2 x + 7}$$

$$SSS :=, [1.406545762]$$

$$Orb =, [1.406545762, 1.406545762, 1.406545762, 1.406545762, 1.406545762, 1.406545762, 1.406545762, 1.406545762, 1.406545762, 1.406545762, 1.406545762, 1.406545762, 1.406545762, 1.406545762, 1.406545762, 1.406545762, 1.406545762, 1.406545762]$$

$$x0 =, -10.1$$

$$f =, \frac{4 x^3 + 5 x^2 + 7 x + 5}{2 x^3 + 10 x^2 + 8 x + 6}$$

$$SSS :=, [0.7994227258]$$

$$Orb =, [0.7994227260, 0.7994227260, 0.7994227260, 0.7994227260, 0.7994227260, 0.7994227260, 0.7994227260, 0.7994227260, 0.7994227260, 0.7994227260, 0.7994227260, 0.7994227260, 0.7994227260, 0.7994227260, 0.7994227260, 0.7994227260, 0.7994227260, 0.7994227260]$$

$$x0 =, -10.1$$

$$f =, \frac{8 x^3 + 4 x^2 + 8 x + 6}{10 x^3 + 4 x^2 + 9 x + 7}$$

$$SSS :=, [0.8696205581]$$

$$Orb =, [0.8696205577, 0.8696205577, 0.8696205577, 0.8696205577, 0.8696205577, 0.8696205577, 0.8696205577, 0.8696205577, 0.8696205577, 0.8696205577, 0.8696205577, 0.8696205577, 0.8696205577, 0.8696205577, 0.8696205577, 0.8696205577, 0.8696205577, 0.8696205577]$$

$$x0 =, -10.1$$

$$f =, \frac{x^3 + 9 x^2 + 2 x + 3}{4 x^3 + 2 x^2 + 5 x + 1}$$

$$SSS :=, [1.197066008, -1.260488554]$$

$$Orb =, [1.197066008, 1.197066007, 1.197066008, 1.197066007, 1.197066008, 1.197066007, 1.197066008, 1.197066007, 1.197066008, 1.197066007, 1.197066008, 1.197066007, 1.197066008, 1.197066007, 1.197066008, 1.197066007, 1.197066008, 1.197066007]$$

$$x0 =, 10.1$$

$$f =, \frac{3x^3 + 3x^2 + 7x + 7}{10x^3 + 8x^2 + 3x + 5}$$

$$SSS :=, [0.8620096788]$$

Orb =, [0.8620096794, 0.8620096783, 0.8620096794, 0.8620096783, 0.8620096794,
0.8620096783, 0.8620096794, 0.8620096783, 0.8620096794, 0.8620096783, 0.8620096794,
0.8620096783, 0.8620096794, 0.8620096783, 0.8620096794, 0.8620096783, 0.8620096794,
0.8620096783, 0.8620096794, 0.8620096783, 0.8620096794]

$$x0 =, 10.1$$

$$f =, \frac{3x^3 + x^2 + x + 3}{10x^3 + 10x^2 + 5x + 5}$$

$$SSS :=, [0.4042380413]$$

Orb =, [0.4042380416, 0.4042380414, 0.4042380416, 0.4042380414, 0.4042380416,
0.4042380414, 0.4042380416, 0.4042380414, 0.4042380416, 0.4042380414, 0.4042380416,
0.4042380414, 0.4042380416, 0.4042380414, 0.4042380416, 0.4042380414, 0.4042380416,
0.4042380414, 0.4042380416, 0.4042380414, 0.4042380416]

$$x0 =, 10.1$$

$$f =, \frac{4x^3 + 8x^2 + 2x + 10}{9x^3 + 5x^2 + 9x + 2}$$

$$SSS :=, [0.9747201978]$$

Orb =, [0.9747201986, 0.9747201970, 0.9747201986, 0.9747201970, 0.9747201986,
0.9747201970, 0.9747201986, 0.9747201970, 0.9747201986, 0.9747201970, 0.9747201986,
0.9747201970, 0.9747201986, 0.9747201970, 0.9747201986, 0.9747201970, 0.9747201986,
0.9747201970, 0.9747201986, 0.9747201970, 0.9747201986]

$$x0 =, 10.1$$

$$f =, \frac{10x^3 + 3x^2 + 4x + 7}{9x^3 + 4x^2 + 10x + 9}$$

$$SSS :=, [0.6982012560]$$

Orb =, [0.6982012561, 0.6982012562, 0.6982012559, 0.6982012561, 0.6982012562,
0.6982012559, 0.6982012561, 0.6982012562, 0.6982012559, 0.6982012561, 0.6982012562,
0.6982012559, 0.6982012561, 0.6982012562, 0.6982012559, 0.6982012561, 0.6982012562,
0.6982012559, 0.6982012561, 0.6982012562, 0.6982012559]

$$x0 =, 10.1$$

$$f =, \frac{x^3 + 9x^2 + 8x + 7}{7x^3 + 4x^2 + 2x + 10}$$

$$SSS :=, [1.070745313]$$

$$Orb =, [1.070745313, 1.070745313, 1.070745313, 1.070745313, 1.070745313, 1.070745313, 1.070745313, 1.070745313, 1.070745313, 1.070745313, 1.070745313, 1.070745313, 1.070745313, 1.070745313, 1.070745313, 1.070745313, 1.070745313, 1.070745313]$$

$$x0 =, 10.1$$

$$f =, \frac{4x^3 + 6x^2 + x + 8}{9x^3 + 2x^2 + 7x + 1}$$

$$SSS :=, [1.]$$

$$Orb =, [1.000000001, 0.999999989, 1.000000001, 0.999999989, 1.000000001, 0.999999989, 1.000000001, 0.999999989, 1.000000001, 0.999999989, 1.000000001, 0.999999989, 1.000000001, 0.999999989, 1.000000001, 0.999999989, 1.000000001, 1.000000001]$$

$$x0 =, 10.1$$

$$f =, \frac{4x^3 + 9x^2 + 9x + 5}{4x^3 + 6x^2 + 4x + 3}$$

$$SSS :=, [1.460448809]$$

$$Orb =, [1.460448809, 1.460448809, 1.460448809, 1.460448809, 1.460448809, 1.460448809, 1.460448809, 1.460448809, 1.460448809, 1.460448809, 1.460448809, 1.460448809, 1.460448809, 1.460448809, 1.460448809, 1.460448809, 1.460448809, 1.460448809]$$

$$x0 =, 10.1$$

$$f =, \frac{2x^3 + 8x^2 + 10x + 4}{3x^3 + 5x^2 + 2x + 7}$$

$$SSS :=, [1.406545762]$$

$$Orb =, [1.406545762, 1.406545762, 1.406545762, 1.406545762, 1.406545762, 1.406545762, 1.406545762, 1.406545762, 1.406545762, 1.406545762, 1.406545762, 1.406545762, 1.406545762, 1.406545762, 1.406545762, 1.406545762, 1.406545762, 1.406545762]$$

$$x0 =, 10.1$$

$$f =, \frac{4x^3 + 5x^2 + 7x + 5}{2x^3 + 10x^2 + 8x + 6}$$

$$SSS :=, [0.7994227258]$$

Orb =, [0.7994227260, 0.7994227260, 0.7994227260, 0.7994227260, 0.7994227260,
0.7994227260, 0.7994227260, 0.7994227260, 0.7994227260, 0.7994227260, 0.7994227260,
0.7994227260, 0.7994227260, 0.7994227260, 0.7994227260, 0.7994227260, 0.7994227260,
0.7994227260, 0.7994227260, 0.7994227260, 0.7994227260]

$$x0 =, 10.1$$

$$f =, \frac{8x^3 + 4x^2 + 8x + 6}{10x^3 + 4x^2 + 9x + 7}$$

$$SSS :=, [0.8696205581]$$

Orb =, [0.8696205577, 0.8696205577, 0.8696205577, 0.8696205577, 0.8696205577,
0.8696205577, 0.8696205577, 0.8696205577, 0.8696205577, 0.8696205577, 0.8696205577,
0.8696205577, 0.8696205577, 0.8696205577, 0.8696205577, 0.8696205577, 0.8696205577,
0.8696205577, 0.8696205577, 0.8696205577, 0.8696205577]

$$x0 =, 10.1$$

$$f =, \frac{x^3 + 9x^2 + 2x + 3}{4x^3 + 2x^2 + 5x + 1}$$

$$SSS :=, [1.197066008, -1.260488554]$$

Orb =, [1.197066008, 1.197066007, 1.197066008, 1.197066007, 1.197066008, 1.197066007,
1.197066008, 1.197066007, 1.197066008, 1.197066007, 1.197066008, 1.197066007,
1.197066008, 1.197066007, 1.197066008, 1.197066007, 1.197066008, 1.197066007,
1.197066008, 1.197066007, 1.197066008]

$$x0 =, 100.1$$

$$f =, \frac{3x^3 + 3x^2 + 7x + 7}{10x^3 + 8x^2 + 3x + 5}$$

$$SSS :=, [0.8620096788]$$

Orb =, [0.8620096794, 0.8620096783, 0.8620096794, 0.8620096783, 0.8620096794,
0.8620096783, 0.8620096794, 0.8620096783, 0.8620096794, 0.8620096783, 0.8620096794,
0.8620096783, 0.8620096794, 0.8620096783, 0.8620096794, 0.8620096783, 0.8620096794,
0.8620096783, 0.8620096794, 0.8620096783, 0.8620096794]

$$x0 =, 100.1$$

$$f =, \frac{3x^3 + x^2 + x + 3}{10x^3 + 10x^2 + 5x + 5}$$

$$SSS :=, [0.4042380413]$$

Orb =, [0.4042380416, 0.4042380414, 0.4042380416, 0.4042380414, 0.4042380416,
0.4042380414, 0.4042380416, 0.4042380414, 0.4042380416, 0.4042380414, 0.4042380416,
0.4042380414, 0.4042380416, 0.4042380414, 0.4042380416, 0.4042380414, 0.4042380416,
0.4042380414, 0.4042380416, 0.4042380414, 0.4042380416]

$$x0 =, 100.1$$

$$f =, \frac{4x^3 + 8x^2 + 2x + 10}{9x^3 + 5x^2 + 9x + 2}$$

$$SSS :=, [0.9747201978]$$

Orb =, [0.9747201982, 0.9747201978, 0.9747201982, 0.9747201978, 0.9747201982,
0.9747201978, 0.9747201982, 0.9747201978, 0.9747201982, 0.9747201978, 0.9747201982,
0.9747201978, 0.9747201982, 0.9747201978, 0.9747201982, 0.9747201978, 0.9747201982,
0.9747201978, 0.9747201982, 0.9747201978, 0.9747201982]

$$x0 =, 100.1$$

$$f =, \frac{10x^3 + 3x^2 + 4x + 7}{9x^3 + 4x^2 + 10x + 9}$$

$$SSS :=, [0.6982012560]$$

Orb =, [0.6982012559, 0.6982012561, 0.6982012562, 0.6982012559, 0.6982012561,
0.6982012562, 0.6982012559, 0.6982012561, 0.6982012562, 0.6982012559, 0.6982012561,
0.6982012562, 0.6982012559, 0.6982012561, 0.6982012562, 0.6982012559, 0.6982012561,
0.6982012562, 0.6982012559, 0.6982012561, 0.6982012562]

$$x0 =, 100.1$$

$$f =, \frac{x^3 + 9x^2 + 8x + 7}{7x^3 + 4x^2 + 2x + 10}$$

$$SSS :=, [1.070745313]$$

Orb =, [1.070745313, 1.070745313, 1.070745313, 1.070745313, 1.070745313, 1.070745313,
1.070745313, 1.070745313, 1.070745313, 1.070745313, 1.070745313, 1.070745313,
1.070745313, 1.070745313, 1.070745313, 1.070745313, 1.070745313, 1.070745313,
1.070745313, 1.070745313, 1.070745313]

$$x0 =, 100.1$$

$$f =, \frac{4x^3 + 6x^2 + x + 8}{9x^3 + 2x^2 + 7x + 1}$$

$$SSS :=, [1.]$$

Orb =, [1.000000001, 0.9999999989, 1.000000001, 0.9999999989, 1.000000001, 0.9999999989,
1.000000001, 0.9999999989, 1.000000001, 0.9999999989, 1.000000001, 0.9999999989,
1.000000001, 0.9999999989, 1.000000001, 0.9999999989, 1.000000001, 0.9999999989,
1.000000001, 0.9999999989, 1.000000001]

$$x0 =, 100.1$$

$$f =, \frac{4x^3 + 9x^2 + 9x + 5}{4x^3 + 6x^2 + 4x + 3}$$

$$SSS :=, [1.460448809]$$

Orb =, [1.460448809, 1.460448809, 1.460448809, 1.460448809, 1.460448809, 1.460448809,
1.460448809, 1.460448809, 1.460448809, 1.460448809, 1.460448809, 1.460448809,
1.460448809, 1.460448809, 1.460448809, 1.460448809, 1.460448809, 1.460448809,
1.460448809, 1.460448809, 1.460448809]

$$x0 =, 100.1$$

$$f =, \frac{2x^3 + 8x^2 + 10x + 4}{3x^3 + 5x^2 + 2x + 7}$$

$$SSS :=, [1.406545762]$$

Orb =, [1.406545762, 1.406545762, 1.406545762, 1.406545762, 1.406545762, 1.406545762,
1.406545762, 1.406545762, 1.406545762, 1.406545762, 1.406545762, 1.406545762,
1.406545762, 1.406545762, 1.406545762, 1.406545762, 1.406545762, 1.406545762,
1.406545762, 1.406545762, 1.406545762]

$$x0 =, 100.1$$

$$f =, \frac{4x^3 + 5x^2 + 7x + 5}{2x^3 + 10x^2 + 8x + 6}$$

$$SSS :=, [0.7994227258]$$

Orb =, [0.7994227260, 0.7994227260, 0.7994227260, 0.7994227260, 0.7994227260,
0.7994227260, 0.7994227260, 0.7994227260, 0.7994227260, 0.7994227260, 0.7994227260,
0.7994227260, 0.7994227260, 0.7994227260, 0.7994227260, 0.7994227260, 0.7994227260,
0.7994227260, 0.7994227260, 0.7994227260]

$$x0 =, 100.1$$

$$f = \frac{8x^3 + 4x^2 + 8x + 6}{10x^3 + 4x^2 + 9x + 7}$$

$$SSS :=, [0.8696205581]$$

$Orb = [0.8696205577, 0.8696205577, 0.8696205577, 0.8696205577, 0.8696205577,$
 $0.8696205577, 0.8696205577, 0.8696205577, 0.8696205577, 0.8696205577, 0.8696205577,$
 $0.8696205577, 0.8696205577, 0.8696205577, 0.8696205577, 0.8696205577, 0.8696205577,$
 $0.8696205577, 0.8696205577, 0.8696205577, 0.8696205577]$

$$x0 = , 100.1$$

$$f = \frac{x^3 + 9x^2 + 2x + 3}{4x^3 + 2x^2 + 5x + 1}$$

$$SSS :=, [1.197066008, -1.260488554]$$

$Orb = [1.197066008, 1.197066007, 1.197066008, 1.197066007, 1.197066008, 1.197066007,$
 $1.197066008, 1.197066007, 1.197066008, 1.197066007, 1.197066008, 1.197066007,$
 $1.197066008, 1.197066007, 1.197066008, 1.197066007, 1.197066008, 1.197066007,$
 $1.197066008, 1.197066007, 1.197066008]$

(3)

```
> for k in {1, 2, 2.5, 3.1, 3.2, 3.3, 3.5} do
```

$$f := k \cdot x \cdot (1 - x);$$

```
print( `k =`, k, `orbit =`, Orb( f, x, 0.5, 200, 230 ) ); end;
```

$k = 1, orbit = , [0.004828613965, 0.004805298452, 0.004782207559, 0.004759338050,$
 $0.004736686752, 0.004714250550, 0.004692026392, 0.004670011280, 0.004648202275,$
 $0.004626596490, 0.004605191095, 0.004583983310, 0.004562970407, 0.004542149708,$
 $0.004521518584, 0.004501074454, 0.004480814783, 0.004460737082, 0.004440838907,$
 $0.004421117857, 0.004401571574, 0.004382197742, 0.004362994085, 0.004343958368,$
 $0.004325088394, 0.004306382004, 0.004287837078, 0.004269451531, 0.004251223315,$
 $0.004233150415, 0.004215230853]$

$$k = 2, \text{orbit} = [0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, \\ 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, \\ 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, \\ 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, \\ 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, \\ 0.5000000000]$$
$$k = , 2.5, orbit = , [0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000,$$

$$0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000,$$

$$0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000,$$

0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000,
0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000,
0.6000000000, 0.6000000000]

$k = 3.1$, $orbit =$, [0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245,
0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245,
0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245,
0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245,
0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245,
0.7645665203, 0.5580141245]

$k = 3.2$, $orbit =$, [0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091,
0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091,
0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091,
0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091,
0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091,
0.7994554906, 0.5130445091]

$k = 3.3$, $orbit =$, [0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198,
0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198,
0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198,
0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198,
0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198,
0.8236032832, 0.4794270198]

$k = 3.5$, $orbit =$, [0.5008842111, 0.8749972637, 0.3828196827, 0.8269407062, 0.5008842111, (4)
0.8749972637, 0.3828196827, 0.8269407062, 0.5008842111, 0.8749972637, 0.3828196827,
0.8269407062, 0.5008842111, 0.8749972637, 0.3828196827, 0.8269407062, 0.5008842111,
0.8749972637, 0.3828196827, 0.8269407062, 0.5008842111, 0.8749972637, 0.3828196827,
0.8269407062, 0.5008842111, 0.8749972637, 0.3828196827, 0.8269407062, 0.5008842111,
0.8749972637, 0.3828196827]

> '2000 to 2030 was too much for maple!'
2000 to 2030 was too much for maple! (5)

> 'for 1..2.5 it was stable, but at 3..3.3 it alternated between 2 states and at 3.5 it alternated
between 3 states'
for 1..2.5 it was stable, but at 3..3.3 it alternated between 2 states and at 3.5 it alternated between 3 (6)
states

> for k from 355 to 400 do $f := \text{MakeNicePol}([1, 2, 3, 4, 5, 6, 7, 8], k, x) : sss := \text{evalf}(SSS(f,$

$$\begin{aligned}
 & x)) : \text{if } nops(sss) > 1 \text{ then } print(k, f, sss); \text{break; end end:} \\
 & 361, \frac{1}{361} x^9 - \frac{36}{361} x^8 + \frac{546}{361} x^7 - \frac{4536}{361} x^6 + \frac{22449}{361} x^5 - \frac{67284}{361} x^4 + \frac{118124}{361} x^3 \\
 & - \frac{109584}{361} x^2 + \frac{40681}{361} x, [3., 5.]
 \end{aligned} \tag{7}$$

> `361 is the smallest k value with 2 stable steady states`
361 is the smallest k value with 2 stable steady states

> Orb(MakeNicePol([1, 2, 3, 4, 5, 6, 7, 8], 361, x), x, 5.4, 10, 15)
 [5.0921758, 4.9027934, 5.0858549, 4.9096827, 5.0807794, 4.9153429]

> for k from 2500 to 10000 do f := MakeNicePol([1, 2, 3, 4, 5, 6, 7, 8], k, x) : sss :=
 evalf(SSS(f, x)) : if nops(sss) > 3 then print(k, f, sss); break; end end:

$$\begin{aligned}
 & 2521, \frac{1}{2521} x^9 - \frac{36}{2521} x^8 + \frac{546}{2521} x^7 - \frac{4536}{2521} x^6 + \frac{22449}{2521} x^5 - \frac{67284}{2521} x^4 + \frac{118124}{2521} x^3 \\
 & - \frac{109584}{2521} x^2 + \frac{42841}{2521} x, [1., 3., 5., 7.]
 \end{aligned} \tag{10}$$

> Orb(MakeNicePol([1, 2, 3, 4, 5, 6, 7, 8], 2521, x), x, 6.8, 10, 15)
 [6.9403929, 7.0482248, 6.9442747, 7.0458398, 6.9475589, 7.0436533]

> `looks good to me!`