Homework for Lecture 8 of Dr. Z.'s Dynamical Models in Biology class

Email the answers (as .pdf file) to

ShaloshBEkhad@gmail.com

by 8:00pm Monday, Oct. 6,, 2021.

Subject: hw8

with an attachment hw8FirstLast.pdf and/or hw8FirstLast.txt

Using

`http://sites.math.rutgers.edu/~zeilberg/Bio25/DMB8.txt`

1. Find ten random rational functions using `raRF(x,3)` .

For each of them find all the stable steady-states.

Then using `Orb(f,x,x0,2000,2020);` for `x0=-100.1, -10.1, 10.1, 100.1`

verify that the orbit always converges to that stable steady-state.

2. Find the terms between 2000-th and 2030-th terms of the **orbit** starting at $x_0 = 0.5$ of the recurrence

$$x_n = kx_{n-1}(1 - x_{n-1}) \quad ,$$

for $k = 1$, $k = 2$, $k = 2.5$, $k = 3.1$, $k = 3.2$, $k = 3.3$, $k = 3.5$. What do you find?

3. By Playing "High-Low" find the smallest positive integer such that `k`,

```
f:=MakeNicePol([1,2,3,4,5,6,7,8],k,x);
```

has two stable steady-states. Pick random `x0` from their neighborhood and verify that indeed the orbits converges to them.

Then keep upping k to find the smallest integer such that it has four stable steady-states. Pick random `x0` from their neighborhood and verify that indeed the orbits converges to them.

problem #1

```
> f := raRF(x, 3)
```

1 $f := \frac{7x^3 + x^2 + 4x + 8}{4x^3 + 8x^2 + 6x + 1}$ converges to 1.0367

```
> SSS(f, x)
```

$[RootOf(4\_Z^4 + \_Z^3 + 5\_Z^2 - 3\_Z - 8, index = 1)]$

```
> Orb(f, x, -100.1, 2000, 2020)
[1.036765162, 1.036765162, 1.036765162, 1.036765162, 1.036765162, 1.036765162, 1.036765162, 1.036765162, 1.036765162, 1.036765162, 1.036765162, 1.036765162, 1.036765162,
 1.036765162, 1.036765162, 1.036765162, 1.036765162, 1.036765162, 1.036765162, 1.036765162]
> Orb(f, x, -10.1, 2000, 2020)
[1.036765162, 1.036765162, 1.036765162, 1.036765162, 1.036765162, 1.036765162, 1.036765162, 1.036765162, 1.036765162, 1.036765162, 1.036765162, 1.036765162, 1.036765162,
 1.036765162, 1.036765162, 1.036765162, 1.036765162, 1.036765162, 1.036765162, 1.036765162]
> Orb(f, x, 10.1, 2000, 2020)
[1.036765162, 1.036765162, 1.036765162, 1.036765162, 1.036765162, 1.036765162, 1.036765162, 1.036765162, 1.036765162, 1.036765162, 1.036765162, 1.036765162, 1.036765162,
 1.036765162, 1.036765162, 1.036765162, 1.036765162, 1.036765162, 1.036765162, 1.036765162]
> Orb(f, x, 100.1, 2000, 2020)
[1.036765162, 1.036765162, 1.036765162, 1.036765162, 1.036765162, 1.036765162, 1.036765162, 1.036765162, 1.036765162, 1.036765162, 1.036765162, 1.036765162, 1.036765162,
 1.036765162, 1.036765162, 1.036765162, 1.036765162, 1.036765162, 1.036765162, 1.036765162]
```

```
> f := raRF(x, 3)
```

2 $f := \frac{10x^3 + 10x^2 + 4x + 1}{10x^3 + 6x^2 + 2x + 3}$ converges to 1.19669

```
> SSS(f, x)
```

$[RootOf(10\_Z^4 - 4\_Z^3 - 8\_Z^2 - \_Z - 1, index = 1)]$

```
> Orb(f, x, -100.1, 2000, 2020)
[1.196690922, 1.196690923, 1.196690922, 1.196690923, 1.196690922, 1.196690923, 1.196690922, 1.196690923, 1.196690922, 1.196690923, 1.196690922, 1.196690923, 1.196690922,
 1.196690923, 1.196690922, 1.196690923, 1.196690922, 1.196690923, 1.196690922, 1.196690923]
> Orb(f, x, -10.1, 2000, 2020)
[1.196690922, 1.196690923, 1.196690922, 1.196690923, 1.196690922, 1.196690923, 1.196690922, 1.196690923, 1.196690922, 1.196690923, 1.196690922, 1.196690923, 1.196690922,
 1.196690923, 1.196690922, 1.196690923, 1.196690922, 1.196690923, 1.196690922, 1.196690923]
> Orb(f, x, 10.1, 2000, 2020)
[1.196690922, 1.196690923, 1.196690922, 1.196690923, 1.196690922, 1.196690923, 1.196690922, 1.196690923, 1.196690922, 1.196690923, 1.196690922, 1.196690923, 1.196690922,
 1.196690923, 1.196690922, 1.196690923, 1.196690922, 1.196690923, 1.196690922, 1.196690923]
> Orb(f, x, 100.1, 2000, 2020)
[1.196690922, 1.196690923, 1.196690922, 1.196690923, 1.196690922, 1.196690923, 1.196690922, 1.196690923, 1.196690922, 1.196690923, 1.196690922, 1.196690923, 1.196690922,
 1.196690923, 1.196690922, 1.196690923, 1.196690922, 1.196690923, 1.196690922, 1.196690923]
```

```
> f := raRF(x, 3)
```

3 $f := \frac{x^3 + 7x^2 + 6x + 1}{10x^3 + 8x^2 + 9x + 2}$ converges to 0.591035

```
> SSS(f, x)
```

$[RootOf(10\_Z^4 + 7\_Z^3 + 2\_Z^2 - 4\_Z - 1, index = 1)]$

```
> Orb(f, x, -100.1, 2000, 2020)
[0.5910351320, 0.5910351310, 0.5910351311, 0.5910351312, 0.5910351313, 0.5910351314, 0.5910351320, 0.5910351310, 0.5910351311, 0.5910351312, 0.5910351313, 0.5910351314,
 0.5910351320, 0.5910351310, 0.5910351311, 0.5910351312, 0.5910351313, 0.5910351314, 0.5910351320, 0.5910351310, 0.5910351311]
> Orb(f, x, -10.1, 2000, 2020)
[0.5910351312, 0.5910351313, 0.5910351314, 0.5910351320, 0.5910351310, 0.5910351311, 0.5910351312, 0.5910351313, 0.5910351314, 0.5910351320, 0.5910351310, 0.5910351311,
 0.5910351312, 0.5910351313, 0.5910351314, 0.5910351320, 0.5910351310, 0.5910351311, 0.5910351312, 0.5910351313, 0.5910351314]
> Orb(f, x, 10.1, 2000, 2020)
[0.5910351312, 0.5910351313, 0.5910351314, 0.5910351320, 0.5910351310, 0.5910351311, 0.5910351312, 0.5910351313, 0.5910351314, 0.5910351320, 0.5910351310, 0.5910351311,
 0.5910351312, 0.5910351313, 0.5910351314, 0.5910351320, 0.5910351310, 0.5910351311, 0.5910351312, 0.5910351313, 0.5910351314]
> Orb(f, x, 100.1, 2000, 2020)
[0.5910351314, 0.5910351320, 0.5910351310, 0.5910351311, 0.5910351312, 0.5910351313, 0.5910351314, 0.5910351320, 0.5910351310, 0.5910351311, 0.5910351312, 0.5910351313,
 0.5910351314, 0.5910351320, 0.5910351310, 0.5910351311, 0.5910351312, 0.5910351313, 0.5910351314, 0.5910351320, 0.5910351310]
```

```
> f := raRF(x, 3)
```

4 $f := \frac{7x^3 + 7x^2 + 10x + 1}{5x^3 + 3x^2 + 10x + 5}$ converges to 1.157

```
> SSS(f, x)
```

$[RootOf(5\_Z^4 - 4\_Z^3 + 3\_Z^2 - 5\_Z - 1, index = 1)]$

```
> Orb(f, x, -100.1, 2000, 2020)
[1.157261819, 1.157261818, 1.157261819, 1.157261818, 1.157261819, 1.157261818, 1.157261819, 1.157261818, 1.157261819, 1.157261818, 1.157261819, 1.157261818, 1.157261819,
 1.157261818, 1.157261819, 1.157261818, 1.157261819, 1.157261818, 1.157261819, 1.157261819]
> Orb(f, x, -10.1, 2000, 2020)
[1.157261818, 1.157261819, 1.157261818, 1.157261819, 1.157261818, 1.157261819, 1.157261818, 1.157261819, 1.157261818, 1.157261819, 1.157261818, 1.157261819, 1.157261818,
 1.157261819, 1.157261818, 1.157261819, 1.157261818, 1.157261819, 1.157261818, 1.157261818]
> Orb(f, x, 10.1, 2000, 2020)
[1.157261818, 1.157261819, 1.157261818, 1.157261819, 1.157261818, 1.157261819, 1.157261818, 1.157261819, 1.157261818, 1.157261819, 1.157261818, 1.157261819, 1.157261818,
 1.157261819, 1.157261818, 1.157261819, 1.157261818, 1.157261819, 1.157261818, 1.157261818]
> Orb(f, x, 100.1, 2000, 2020)
[1.157261818, 1.157261819, 1.157261818, 1.157261819, 1.157261818, 1.157261819, 1.157261818, 1.157261819, 1.157261818, 1.157261819, 1.157261818, 1.157261819, 1.157261818,
 1.157261819, 1.157261818, 1.157261819, 1.157261818, 1.157261819, 1.157261818, 1.157261818]
```

```
> f := raRF(x, 3)
```

5 $f := \frac{9x^3 + 4x^2 + 3x + 5}{x^3 + 5x^2 + 7x + 5}$ conv. to 2.947

```
> SSS(f, x)
```

$$\left[1 + \frac{\sqrt{6 + 2\sqrt{21}}}{2}\right]$$

```
> Orb(f, x, -100.1, 2000, 2020)
[2.947122972, 2.947122972, 2.947122972, 2.947122972, 2.947122972, 2.947122972, 2.947122972, 2.947122972, 2.947122972, 2.947122972, 2.947122972, 2.947122972,
  2.947122972, 2.947122972, 2.947122972, 2.947122972, 2.947122972, 2.947122972, 2.947122972, 2.947122972]
> Orb(f, x, -10.1, 2000, 2020)
[2.947122972, 2.947122972, 2.947122972, 2.947122972, 2.947122972, 2.947122972, 2.947122972, 2.947122972, 2.947122972, 2.947122972, 2.947122972, 2.947122972,
  2.947122972, 2.947122972, 2.947122972, 2.947122972, 2.947122972, 2.947122972, 2.947122972, 2.947122972]
> Orb(f, x, 10.1, 2000, 2020)
[2.947122972, 2.947122972, 2.947122972, 2.947122972, 2.947122972, 2.947122972, 2.947122972, 2.947122972, 2.947122972, 2.947122972, 2.947122972, 2.947122972,
  2.947122972, 2.947122972, 2.947122972, 2.947122972, 2.947122972, 2.947122972, 2.947122972, 2.947122972]
> Orb(f, x, 100.1, 2000, 2020)
[2.947122972, 2.947122972, 2.947122972, 2.947122972, 2.947122972, 2.947122972, 2.947122972, 2.947122972, 2.947122972, 2.947122972, 2.947122972, 2.947122972,
  2.947122972, 2.947122972, 2.947122972, 2.947122972, 2.947122972, 2.947122972, 2.947122972, 2.947122972]
```

```
> f := raRF(x, 3)
```

6 $f := \frac{10x^3 + 8x^2 + 9x + 1}{3x^3 + 7x^2 + 3x + 7}$ conv. to 2.0254

```
> SSS(f, x)
```

$$[RootOf(3\_Z^4 - 3\_Z^3 - 5\_Z^2 - 2\_Z - 1, index = 1)]$$

```
> Orb(f, x, -100.1, 2000, 2020)
[2.025470110, 2.025470110, 2.025470110, 2.025470110, 2.025470110, 2.025470110, 2.025470110, 2.025470110, 2.025470110, 2.025470110, 2.025470110, 2.025470110,
  2.025470110, 2.025470110, 2.025470110, 2.025470110, 2.025470110, 2.025470110, 2.025470110, 2.025470110]
> Orb(f, x, -10.1, 2000, 2020)
[2.025470110, 2.025470110, 2.025470110, 2.025470110, 2.025470110, 2.025470110, 2.025470110, 2.025470110, 2.025470110, 2.025470110, 2.025470110, 2.025470110,
  2.025470110, 2.025470110, 2.025470110, 2.025470110, 2.025470110, 2.025470110, 2.025470110, 2.025470110]
> Orb(f, x, 10.1, 2000, 2020)
[2.025470110, 2.025470110, 2.025470110, 2.025470110, 2.025470110, 2.025470110, 2.025470110, 2.025470110, 2.025470110, 2.025470110, 2.025470110, 2.025470110,
  2.025470110, 2.025470110, 2.025470110, 2.025470110, 2.025470110, 2.025470110, 2.025470110, 2.025470110]
> Orb(f, x, 100.1, 2000, 2020)
[2.025470110, 2.025470110, 2.025470110, 2.025470110, 2.025470110, 2.025470110, 2.025470110, 2.025470110, 2.025470110, 2.025470110, 2.025470110, 2.025470110,
  2.025470110, 2.025470110, 2.025470110, 2.025470110, 2.025470110, 2.025470110, 2.025470110, 2.025470110]
```

```
> f := raRF(x, 3)
```

7 $f := \frac{3x^3 + 10x^2 + 5x + 3}{x^3 + 5x^2 + 9x + 5}$ conv to 1.108

```
> SSS(f, x)
```

$$[RootOf(\_Z^4 + 2\_Z^3 - \_Z^2 - 3, index = 1)]$$

```
> Orb(f, x, -100.1, 2000, 2020)
[1.108000159, 1.108000159, 1.108000159, 1.108000159, 1.108000159, 1.108000159, 1.108000159, 1.108000159, 1.108000159, 1.108000159, 1.108000159, 1.108000159,
  1.108000159, 1.108000159, 1.108000159, 1.108000159, 1.108000159, 1.108000159, 1.108000159, 1.108000159]
> Orb(f, x, -10.1, 2000, 2020)
[1.108000159, 1.108000159, 1.108000159, 1.108000159, 1.108000159, 1.108000159, 1.108000159, 1.108000159, 1.108000159, 1.108000159, 1.108000159, 1.108000159,
  1.108000159, 1.108000159, 1.108000159, 1.108000159, 1.108000159, 1.108000159, 1.108000159, 1.108000159]
> Orb(f, x, 10.1, 2000, 2020)
[1.108000159, 1.108000159, 1.108000159, 1.108000159, 1.108000159, 1.108000159, 1.108000159, 1.108000159, 1.108000159, 1.108000159, 1.108000159, 1.108000159,
  1.108000159, 1.108000159, 1.108000159, 1.108000159, 1.108000159, 1.108000159, 1.108000159, 1.108000159]
> Orb(f, x, 100.1, 2000, 2020)
[1.108000159, 1.108000159, 1.108000159, 1.108000159, 1.108000159, 1.108000159, 1.108000159, 1.108000159, 1.108000159, 1.108000159, 1.108000159, 1.108000159,
  1.108000159, 1.108000159, 1.108000159, 1.108000159, 1.108000159, 1.108000159, 1.108000159, 1.108000159]
```

```
> f := raRF(x, 3)
```

8 $f := \frac{6x^3 + 8x^2 + 2x + 3}{9x^3 + 5x^2 + 8x + 2}$ conv. to 0.78921

```
> SSS(f, x)
```

$$[RootOf(9\_Z^4 - \_Z^3 - 3, index = 1)]$$

```
> Orb(f, x, -100.1, 2000, 2020)
[0.7892134801, 0.7892134801, 0.7892134801, 0.7892134801, 0.7892134801, 0.7892134801, 0.7892134801, 0.7892134801, 0.7892134801, 0.7892134801, 0.7892134801, 0.7892134801,
  0.7892134801, 0.7892134801, 0.7892134801, 0.7892134801, 0.7892134801, 0.7892134801, 0.7892134801, 0.7892134801]
> Orb(f, x, -10.1, 2000, 2020)
[0.7892134801, 0.7892134801, 0.7892134801, 0.7892134801, 0.7892134801, 0.7892134801, 0.7892134801, 0.7892134801, 0.7892134801, 0.7892134801, 0.7892134801, 0.7892134801,
  0.7892134801, 0.7892134801, 0.7892134801, 0.7892134801, 0.7892134801, 0.7892134801, 0.7892134801, 0.7892134801]
> Orb(f, x, 10.1, 2000, 2020)
[0.7892134801, 0.7892134801, 0.7892134801, 0.7892134801, 0.7892134801, 0.7892134801, 0.7892134801, 0.7892134801, 0.7892134801, 0.7892134801, 0.7892134801, 0.7892134801,
  0.7892134801, 0.7892134801, 0.7892134801, 0.7892134801, 0.7892134801, 0.7892134801, 0.7892134801, 0.7892134801]
> Orb(f, x, 100.1, 2000, 2020)
[0.7892134801, 0.7892134801, 0.7892134801, 0.7892134801, 0.7892134801, 0.7892134801, 0.7892134801, 0.7892134801, 0.7892134801, 0.7892134801, 0.7892134801, 0.7892134801,
  0.7892134801, 0.7892134801, 0.7892134801, 0.7892134801, 0.7892134801, 0.7892134801, 0.7892134801, 0.7892134801]
```

```
> f := raRF(x, 3)
```

9 $f := \frac{10x^3 + 10x^2 + 2x + 1}{10x^3 + 6x^2 + 6x + 9}$ conv. to 0.1586

```
> SSS(f, x)
```

$[RootOf(10\_Z^4 - 4\_Z^3 - 4\_Z^2 + 7\_Z - 1, index = 1)]$

```
> Orb(f, x, -100.1, 2000, 2020)
[0.1586082835, 0.1586082835, 0.1586082835, 0.1586082835, 0.1586082835, 0.1586082835, 0.1586082835, 0.1586082835, 0.1586082835, 0.1586082835, 0.1586082835,
 0.1586082835, 0.1586082835, 0.1586082835, 0.1586082835, 0.1586082835, 0.1586082835, 0.1586082835, 0.1586082835, 0.1586082835]
> Orb(f, x, -10.1, 2000, 2020)
[0.1586082835, 0.1586082835, 0.1586082835, 0.1586082835, 0.1586082835, 0.1586082835, 0.1586082835, 0.1586082835, 0.1586082835, 0.1586082835, 0.1586082835,
 0.1586082835, 0.1586082835, 0.1586082835, 0.1586082835, 0.1586082835, 0.1586082835, 0.1586082835, 0.1586082835, 0.1586082835]
> Orb(f, x, 10.1, 2000, 2020)
[0.1586082835, 0.1586082835, 0.1586082835, 0.1586082835, 0.1586082835, 0.1586082835, 0.1586082835, 0.1586082835, 0.1586082835, 0.1586082835, 0.1586082835,
 0.1586082835, 0.1586082835, 0.1586082835, 0.1586082835, 0.1586082835, 0.1586082835, 0.1586082835, 0.1586082835, 0.1586082835]
> Orb(f, x, 100.1, 2000, 2020)
[0.1586082835, 0.1586082835, 0.1586082835, 0.1586082835, 0.1586082835, 0.1586082835, 0.1586082835, 0.1586082835, 0.1586082835, 0.1586082835, 0.1586082835,
 0.1586082835, 0.1586082835, 0.1586082835, 0.1586082835, 0.1586082835, 0.1586082835, 0.1586082835, 0.1586082835, 0.1586082835]
```

```
> f := raRF(x, 3)
```

10 $f := \frac{8x^3 + 9x^2 + 4x + 9}{8x^3 + 6x^2 + 7x + 8}$ conv. to 1.036370

```
> SSS(f, x)
```

$[RootOf(8\_Z^4 - 2\_Z^3 - 2\_Z^2 + 4\_Z - 9, index = 1)]$

```
> Orb(f, x, -100.1, 2000, 2020)
[1.036370401, 1.036370401, 1.036370401, 1.036370401, 1.036370401, 1.036370401, 1.036370401, 1.036370401, 1.036370401, 1.036370401, 1.036370401, 1.036370401,
 1.036370401, 1.036370401, 1.036370401, 1.036370401, 1.036370401, 1.036370401, 1.036370401, 1.036370401]
> Orb(f, x, -10.1, 2000, 2020)
[1.036370401, 1.036370401, 1.036370401, 1.036370401, 1.036370401, 1.036370401, 1.036370401, 1.036370401, 1.036370401, 1.036370401, 1.036370401, 1.036370401,
 1.036370401, 1.036370401, 1.036370401, 1.036370401, 1.036370401, 1.036370401, 1.036370401, 1.036370401]
> Orb(f, x, 10.1, 2000, 2020)
[1.036370401, 1.036370401, 1.036370401, 1.036370401, 1.036370401, 1.036370401, 1.036370401, 1.036370401, 1.036370401, 1.036370401, 1.036370401, 1.036370401,
 1.036370401, 1.036370401, 1.036370401, 1.036370401, 1.036370401, 1.036370401, 1.036370401, 1.036370401]
> Orb(f, x, 100.1, 2000, 2020)
[1.036370401, 1.036370401, 1.036370401, 1.036370401, 1.036370401, 1.036370401, 1.036370401, 1.036370401, 1.036370401, 1.036370401, 1.036370401, 1.036370401,
 1.036370401, 1.036370401, 1.036370401, 1.036370401, 1.036370401, 1.036370401, 1.036370401, 1.036370401]
```

PROB. #2

```
> Orb(1 x (1 − x), x, 0.5, 2000, 2030)
[0.0004976675995, 0.0004974199265, 0.0004971724999, 0.0004969253194, 0.0004966783846, 0.0004964316952, 0.0004961852508, 0.0004959390510, 0.0004956930954,
 0.0004954473838, 0.0004952019157, 0.0004949566908, 0.0004947117087, 0.0004944669690, 0.0004942224714, 0.0004939782155, 0.0004937342010, 0.0004934904275,
 0.0004932468947, 0.0004930036022, 0.0004927605496, 0.0004925177367, 0.0004922751630, 0.0004920328281, 0.0004917907318, 0.0004915488737, 0.0004913072534,
 0.0004910658706, 0.0004908247249, 0.0004905838160, 0.0004903431435]

> Orb(2 x (1 − x), x, 0.5, 2000, 2030)
[0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000,
 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000,
 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000]

> Orb(2.5 x (1 − x), x, 0.5, 2000, 2030)
[0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000,
 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000,
 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000]
```

K=1

K=2

K=2.5

K=1, 2, 2.5 all the orbital values are the same so these are S.S. (stable)

```
> Orb(3.1 x (1 − x), x, 0.5, 2000, 2030)
[0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203,
 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203,
 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245]

> Orb(3.2 x (1 − x), x, 0.5, 2000, 2030)
[0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906,
 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906,
 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091]

> Orb(3.3 x (1 − x), x, 0.5, 2000, 2030)
[0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832,
 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832,
 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198]

> Orb(3.5 x (1 − x), x, 0.5, 2000, 2030)
[0.5008842111, 0.8749972637, 0.3828196827, 0.8269407062, 0.5008842111, 0.8749972637, 0.3828196827, 0.8269407062, 0.5008842111, 0.8749972637, 0.3828196827, 0.8269407062,
 0.5008842111, 0.8749972637, 0.3828196827, 0.8269407062, 0.5008842111, 0.8749972637, 0.3828196827, 0.8269407062, 0.5008842111, 0.8749972637, 0.3828196827, 0.8269407062,
 0.5008842111, 0.8749972637, 0.3828196827, 0.8269407062, 0.5008842111, 0.8749972637, 0.3828196827]
```

K=3.1

K=3.2

K=3.3

K=3.4

When K=3.1, 3.2, 3.3, 3.4 the orbitals bounce between 2 numbers so its unstable S.S.

PROB #3 : tested multiple integers for both cases below.

```
> f := MakeNicePol([1, 2, 3, 4, 5, 6, 7, 8], 361, x)
        f := 1/361 x^8 − 36/361 x^7 + 546/361 x^6 − 4536/361 x^5 + 22449/361 x^4 − 67284/361 x^3 + 118124/361 x^2 − 109584/361 x + 40681/361
> SSS(f, x)
        [3, 5]
> Orb(f, x, 3.5, 1000, 1010)
        [3.0003042, 2.9997023, 3.0003023, 2.9996972, 3.0003126, 2.9996899, 3.0003070, 2.9996928, 3.0003148, 2.9996848, 3.0003143]
> Orb(f, x, 5.1, 1000, 1010)
        [5.0005525, 4.9994615, 5.0005980, 4.9994259, 5.0006803, 4.9993623, 5.0005312, 4.9993682, 5.0006491, 4.9992884, 5.0006114]
```

Smallest value w/ 2 SSS values is 361

```
> f := MakeNicePol([1, 2, 3, 4, 5, 6, 7, 8], 2521, x)
        f := 1/2521 x^8 − 36/2521 x^7 + 546/2521 x^6 − 4536/2521 x^5 + 22449/2521 x^4 − 67284/2521 x^3 + 118124/2521 x^2 − 109584/2521 x + 42841/2521 x
> SSS(f, x)
        [1, 3, 5, 7]
> Orb(f, x, 1.1, 1000, 1010)
        [1.00446666, 0.99560040, 1.00445773, 0.99560910, 1.00444881, 0.99561775, 1.00443992, 0.99562636, 1.00443108, 0.99563496, 1.00442225]
> Orb(f, x, 3.1, 1000, 1010)
        [2.99999944, 2.99999942, 2.99999928, 2.99999940, 2.99999914, 2.99999952, 2.99999988, 3.00000010, 3.00000054, 2.99999921, 2.99999851]
> Orb(f, x, 5.1, 1000, 1010)
        [4.99998819, 5.00000686, 4.99999614, 4.99999196, 5.00000793, 4.99999232, 4.99998305, 4.99999152, 4.99998645, 4.99997930, 4.99997679]
> Orb(f, x, 7.1, 1000, 1010)
        [7.0092564, 6.9905882, 7.0092033, 6.9905529, 7.0092564, 6.9905882, 7.0092033, 6.9905529, 7.0092564, 6.9905882, 7.0092033]
```

Smallest value w/ 4 SSS values is 2521