

Homework for Lecture 8 of Dr. Z.'s Dynamical Models in Biology class

Email the answers (as .pdf file) to

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by 8:00pm Monday, Oct. 6,, 2021.

Subject: hw8

with an attachment hw8FirstLast.pdf and/or hw8FirstLast.txt

Using

<http://sites.math.rutgers.edu/~zeilberg/Bio25/DMB8.txt>

1. Find ten random rational functions using `raRF(x,3)` .

For each of them find all the stable steady-states.

Then using `Orb(f,x,x0,2000,2020)`; for $x_0 = -100.1, -10.1, 10.1, 100.1$

verify that the orbit always converges to that stable steady-state.

2. Find the terms between 2000-th and 2030-th terms of the **orbit** starting at $x_0 = 0.5$ of the recurrence

$$x_n = kx_{n-1}(1 - x_{n-1}) \quad ,$$

for $k = 1, k = 2, k = 2.5, k = 3.1, k = 3.2, k = 3.3, k = 3.5$. What do you find?

3. By Playing “High-Low” find the smallest positive integer such that k ,

`f:=MakeNicePol([1,2,3,4,5,6,7,8],k,x)`;

has two stable steady-states. Pick random x_0 from their neighborhood and verify that indeed the orbits converges to them.

Then keep upping k to find the smallest integer such that it has four stable steady-states. Pick random x_0 from their neighborhood and verify that indeed the orbits converges to them.

<pre> > k:= 360 > f:= MakeNicePol([1,2,3,4,5,6,7,8],k,x); f:= 1/360 x^9 - 1/10 x^8 + 91/60 x^7 - 63/5 x^6 + 7483/120 x^5 - 1869/10 x^4 + 29531/90 x^3 - 1522/5 x^2 + 113 x > evalf(SSS(f,x)); [] </pre>	<pre> > k:= 2520 > f:= MakeNicePol([1,2,3,4,5,6,7,8],k,x); f:= 1/2520 x^9 - 1/70 x^8 + 13/60 x^7 - 9/5 x^6 + 1069/120 x^5 - 267/10 x^4 + 29531/630 x^3 - 1522/35 x^2 + 17 x > evalf(SSS(f,x)); [3., 5.] </pre>
<pre> > k:= 361 > f:= MakeNicePol([1,2,3,4,5,6,7,8],k,x); f:= 1/361 x^9 - 36/361 x^8 + 546/361 x^7 - 4536/361 x^6 + 22449/361 x^5 - 67284/361 x^4 + 118124/361 x^3 - 109584/361 x^2 + 40681/361 x > evalf(SSS(f,x)); [3., 5.] </pre>	<pre> > k:= 2521 > f:= MakeNicePol([1,2,3,4,5,6,7,8],k,x); f:= 1/2521 x^9 - 36/2521 x^8 + 546/2521 x^7 - 4536/2521 x^6 + 22449/2521 x^5 - 67284/2521 x^4 + 118124/2521 x^3 - 109584/2521 x^2 + 42841/2521 x > evalf(SSS(f,x)); [1., 3., 5., 7.] </pre>


```

> Orb(f,x,0.9,1,20);
[1.12790236, 0.92681807, 1.08756149, 0.93912255, 1.07051559, 0.94726242, 1.05975108, 0.95331796, 1.05200886,
 0.95811452, 1.04603783, 0.96206700, 1.04122507, 0.96541365, 1.03722611, 0.96830393, 1.03382870, 0.97083763,
 1.03089330, 0.97308465]
> Orb(f,x,3.1,1,20);
[3.07350158, 3.05366211, 3.03899530, 3.02824583, 3.02041482, 3.01473066, 3.01061754, 3.00764727, 3.00550372,
 3.00396074, 3.00284961, 3.00205022, 3.00147405, 3.00106095, 3.00076296, 3.00054786, 3.00039356, 3.00028269,
 3.00020400, 3.00014572]
> Orb(f,x,4.9,1,20);
[4.92649729, 4.94635294, 4.96102297, 4.97175175, 4.97958483, 4.98527375, 4.98939517, 4.99234761, 4.99448782,
 4.99602818, 4.99715774, 4.99796592, 4.99854096, 4.99894835, 4.99924463, 4.99945749, 4.99961259, 4.99972656,
 4.99979586, 4.99985121]
> Orb(f,x,7.1,1,20);
[6.8720394, 7.0730983, 6.9124934, 7.0608990, 6.9294211, 7.0528019, 6.9402545, 7.0466217, 6.9480272, 7.0417596, 6.9539419,
 7.0379735, 6.9587710, 7.0345282, 6.9630014, 7.0314954, 6.9664177, 7.0289394, 6.9693714, 7.0268867]
        
```

All functions ended up having one stable steady-state
; every orbit went to it

#2

```
> x0 := 0.5;
x0 := 0.5 (2)
> k := 1;
k := 1 (3)
> f := k*x*(1-x);
f := x(1-x) (4)
> L := Orb(f,x,x0,2000,2030);
L := [0.0004976675995, 0.0004974199265, 0.0004971724999, 0.0004969253194, 0.0004966783846, 0.0004964316952, (5)
0.0004961852508, 0.0004959390510, 0.0004956930954, 0.0004954473838, 0.0004952019157, 0.0004949566908,
0.0004947117087, 0.0004944666969, 0.0004942224714, 0.0004939782155, 0.0004937342010, 0.0004934904275,
0.0004932468947, 0.0004930036022, 0.0004927605496, 0.0004925177367, 0.0004922751630, 0.0004920328281,
0.0004917907318, 0.0004915488737, 0.0004913072534, 0.0004910658706, 0.0004908247249, 0.0004905838160,
0.0004903431435]
```

k=1 attracts to ~.0004977

```
> x0 := 0.5;
x0 := 0.5 (2)
> k := 2.5;
k := 2.5 (3)
> f := k*x*(1-x);
f := 2.5*x(1-x) (4)
> L := Orb(f,x,x0,2000,2030);
L := [0.60000000000, 0.60000000000, 0.60000000000, 0.60000000000, 0.60000000000, 0.60000000000, (5)
0.60000000000, 0.60000000000, 0.60000000000, 0.60000000000, 0.60000000000, 0.60000000000,
0.60000000000, 0.60000000000, 0.60000000000, 0.60000000000, 0.60000000000, 0.60000000000,
0.60000000000, 0.60000000000, 0.60000000000, 0.60000000000, 0.60000000000, 0.60000000000,
0.60000000000, 0.60000000000, 0.60000000000]
```

k=2.5 attracts to .6

```
> x0 := 0.5;
x0 := 0.5 (2)
> k := 3.2;
k := 3.2 (3)
> f := k*x*(1-x);
f := 3.2*x(1-x) (4)
> L := Orb(f,x,x0,2000,2030);
L := [0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, (5)
0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906,
0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906,
0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906,
0.5130445091, 0.7994554906, 0.5130445091]
```

k=3.2 attracts/fluctuates to ~.513 ; ~.799

```
> x0 := 0.5;
x0 := 0.5 (2)
> k := 3.5;
k := 3.5 (3)
> f := k*x*(1-x);
f := 3.5*x(1-x) (4)
> L := Orb(f,x,x0,2000,2030);
L := [0.5008842111, 0.8749972637, 0.3828196827, 0.8269407062, 0.5008842111, 0.8749972637, (5)
0.3828196827, 0.8269407062, 0.5008842111, 0.8749972637, 0.3828196827, 0.8269407062,
0.5008842111, 0.8749972637, 0.3828196827, 0.8269407062, 0.5008842111, 0.8749972637,
0.3828196827, 0.8269407062, 0.5008842111, 0.8749972637, 0.3828196827, 0.8269407062,
0.5008842111, 0.8749972637, 0.3828196827]
```

k=3.5 attracts/fluctuates to ~.501, ~.875, ~.383, ; ~.827

```
> x0 := 0.5;
x0 := 0.5 (2)
> k := 2;
k := 2 (3)
> f := k*x*(1-x);
f := 2*x(1-x) (4)
> L := Orb(f,x,x0,2000,2030);
L := [0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, (5)
0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000,
0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000,
0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000,
0.5000000000, 0.5000000000]
```

k=2 attracts to .5

```
> x0 := 0.5;
x0 := 0.5 (2)
> k := 3.1;
k := 3.1 (3)
> f := k*x*(1-x);
f := 3.1*x(1-x) (4)
> L := Orb(f,x,x0,2000,2030);
L := [0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, (5)
0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203,
0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203,
0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203,
0.5580141245, 0.7645665203]
```

k=3.1 attracts/fluctuates to ~.558 ; ~.765

```
> x0 := 0.5;
x0 := 0.5 (2)
> k := 3.3;
k := 3.3 (3)
> f := k*x*(1-x);
f := 3.3*x(1-x) (4)
> L := Orb(f,x,x0,2000,2030);
L := [0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, (5)
0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832,
0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832,
0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832,
0.4794270198, 0.8236032832]
```

k=3.3 attracts/fluctuates to ~.479 ; ~.824