## Homework for Lecture 8 of Dr. Z.'s Dynamical Models in Biology class

Email the answers (as .pdf file) to

ShaloshBEkhad@gmail.com

by 8:00pm Monday, Oct. 6,, 2021.

Subject: hw8

with an attachment hw8FirstLast.pdf and/or hw8FirstLast.txt

Using

http://sites.math.rutgers.edu/~zeilberg/Bio25/DMB8.txt

1. Find ten random rational functions using raRF(x,3).

For each of them find all the stable steady-states.

Then using Orb(f,x,x0,2000,2020); for x0=-100.1, -10.1, 10.1, 100.1

verify that the orbit always converges to that stable steady-state.

**2.** Find the terms between 2000-th and 2030-th terms of the **orbit** starting at  $x_0 = 0.5$  of the recurrence

$$x_n = kx_{n-1}(1 - x_{n-1})$$
 ,

for k = 1, k = 2, k = 2.5, k = 3.1, k = 3.2, k = 3.3, k = 3.5. What do you find?

3. By Playing "High-Low" find the smallest positive integer such that k,

f:=MakeNicePol([1,2,3,4,5,6,7,8],k,x);

has two stable steady-states. Pick random x0 from their neighborhood and verify that indeed the orbits converges to them.

Then keep upping k to find the smallest integer such that it has four stable steady-states. Pick random x0 from their neighborhood and verify that indeed the orbits converges to them.

For each of them find all the stable steady-states. Then using Orb(f,x,x0,2000,2020); for x0=-100.1, -10.1, 10.1, 100.1verify that the orbit always converges to that stable steady-state.  $f := \frac{4x^3 + 2x^2 + x + 7}{2x^3 + 3x^2 + 2x + 2}$  $f := \frac{5x^3 + 3x^2 + 2x + 2}{8x^3 + 5x^2 + 4x + 4}$  $[RootOf(2\_Z^4 - \_Z^3 + \_Z - 7, tndex = 1)]$  $[RootOf(8\_Z^4 + \_Z^2 + 2\_Z - 2, molex = 1)]$ 0.33357197, (0.33537197, 0.35537197, 0.35537197, 0.35537197, 0.35537197, 0.35537197, 0.35537197, 0.3557197, 0.35 OS (C. - 101, 1900, 2020)
| 1437116914, 1437116916, 14 0.5335377397, 0.5335317397, 0.5335377397, 0.5355377397, 0.535577397, 0.555577397, 0.555577397, 0.555577397, 0.555577397, 0.555577397, 0.555577397, 0.555577397, 0.555577397, 0.555577397, 0.555577397, 0.555577397, 0.555 > On-(c, to 1), 2000, 2000 [1437]16814, 1-437116816, 1-43 All or bits converge to 1.437... All orbits converge to 0.534 > 0-64\_(x\_0-100.1,2000,2000) [0.2131-64.6, 0.2131-64.54, 0.2131-64.54, 0.7131-64.54, 0.415328827, 0.4153238827, 0.4153238827, 0.41532882, 0.415328827, 0.415328827, 0.415328827, 0.415328827, 0.41532882, 0.4153282, 0.4153282, 0.41532882, 0.41532882, 0.41532882, 0.4153282, 0.4153282, 0.4153282, 0.4 > 0-0(f\_x 10, 1273-603-7, 0-02 > Or-6(f,x, 100.1, 2000, 2020)
[0.7237546349, 0.7237546349 > Orb(f,x,100.1,2000,2020) [0.4153526827, 0. All or bits converge to 0.415 All orbits converge to 0.7237  $f := \frac{x^3 + 7x^2 + 6x + 6}{2x^3 + 8x^2 + 2x + 4}$  $f = \frac{5x^3 + 10x^2 + 5x + 1}{3x^3 + 10x^2 + 5x + 5}$  $[RootOf(2 Z^4 + 7 Z^8 - 5 Z^2 - 2 Z - 6, index = 1)]$  $[RootOf(3 Z^4 + 5 Z^3 - 5 Z^2 - 1, index = 1)]$ > G-9/f/x-1001, 2000, 2020) [1178008578, 1.1 Orb(f.x.-100.1,2000,2020) [0.84174994031, 0.84174994031, 0.84174994031, 0.847494031, 0.8474994031, 0 0.8474994031, 0.847494031, 0.847494031, 0.847494031, 0.> Or6(fx=10.1,200,202) [1.17806578,1.1780657 0.847999031, 0.8474994031, 0.847494031, All Gross converge to 1.178 All bibits converge to 0.847  $[RootOf(\_Z^4 + \_Z^3 - 3\_Z^2 - 6, inclex = 1)]$  $\left[-\frac{1}{2} + \frac{\sqrt{5}}{2}\right]$ 127(11):12974; 177(18):274; 177 Order 1 - 10.1, 2000, 2000). [ORDINATION OF CREATING AND ASSESSMENT OF CREATING AND ASSESSMENT OF CREATING ASSE > Orb(f<sub>x</sub> [0.1, 2000, 2020) [0.6180339489, 0.6180339889, 0.61803889, 0.61803889, 0.61803889, 0.61803889, 0.61803889, 0.618038989, 0.61803 All orbits converge to 1.772 All orbits converge to 0.618 2. Find the terms between 2000-th and 2030-th terms of the orbit starting at  $x_0 = 0.5$  of the recurrence  $x_n = kx_{n-1}(1 - x_{n-1}) \quad ,$ for k = 1, k = 2, k = 2.5, k = 3.1, k = 3.2, k = 3.3, k = 3.5. What do you find?  $Orb(1 \cdot x \cdot (1-x), x, 0.5, 2000, 2030)$ K=1 0.0004910658706, 0.0004908247249, 0.0004905838160, 0.0004903431435]  $Orb(2 \cdot x \cdot (1 - x), x, 0.5, 2000, 2030)$ K=2 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000]K=7.5 

1. Find ten random rational functions using raRF(x,3).

0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.600000000, 0.600000000, 0.6000000000

| > Orb(3.1 x (1 − x), x, 0.5, 2000, 2030) [0.5580141245, 0.7645665203, 0.5580141245, 0 |  |  |  |  |  |   |   |   |  |   |  |  |                                 |  |  | -3                                     |  |          |      |      |          |                      |             |            |     |            |          |      |             |                     |   |
|---|--|--|--|--|--|---|---|---|--|---|--|--|---------------------------------|--|--|--|--|----------|------|------|----------|----------------------|-------------|------------|-----|------------|----------|------|-------------|---------------------|---|
| > Orb(3)<br>[0.479427<br>0.479  | 3·x·(1<br>0198, 0  | - x),<br>0.8236<br>98, 0.82  | x, 0.5,<br>03283<br>23603:   | 2000, 2<br>2, 0.479<br>2832, 0.  | 030)<br>427019<br>479427                                     | 98, 0.82<br>0198, 0   | 2360328<br>0.82360                                  | 332, 0.4<br>32832,                      | 17942<br>0.479                                   | 70198, 0<br>9427019                               | 0.82360<br>98, 0.82  | )32832,<br>360328                        | 0.47942                         | 70198, 0   | 0.8236<br>8, 0.82                                | 032832                                 | , 0.4794                                   | 270198,  |      |      |          | 4270198,<br>47942701 |             |            | 2,  | K          | 3        | 3    |             |                     |   |
| > Orb(3.<br>[0.500884<br>0.500  | 5·x·(1<br>2111, 0  | - x),<br>0.8749  | x, 0.5,<br>97263   | 2000, 2<br>7, 0.382<br>2637, 0.  | 030)<br>819682<br>382819                                     | 27, 0.82<br>6827, 0   | 2694070<br>0.82694                                  | 062, 0.±                                | 0.500  | 42111,<br>0884211                                 | 0.87499<br>11, 0.87  | 972637,<br>499726                        | 0.38281                         | 96827, 0   | 0.8269<br>7, 0.82                                |  |  |          |      |      |          | 8196827,<br>38281968 |             |            | 52, | K          | :3       | .5   |             |                     |   |
|   |  |  |  | = (,   |  |   |   |   | 5  | all   | 01   | bı                                       | <del>1</del> 01                 | va   | lut  | es                                     | ar   | e ·      | he   | Si   | am       | e s                  | 50          | tre        | 50  | av         | e        |      |             |                     |   |
| 3. By I f:=Mak has two orbits of  | Playing eNiceF stable onverge  | e (final content of the content of t | AVE<br>1-Low'<br>1,2,3,<br>1,2,3,<br>1,2,3,<br>1,2,3,<br>1,2,3,<br>1,2,3,<br>1,2,3,<br>1,2,3,<br>1,2,3,<br>1,2,3,<br>1,2,3,<br>1,2,3,<br>1,2,3,<br>1,2,3,<br>1,2,3,<br>1,2,3,<br>1,2,3,<br>1,2,3,<br>1,2,3,<br>1,2,3,<br>1,2,3,<br>1,2,3,<br>1,2,3,<br>1,2,3,<br>1,2,3,<br>1,2,3,<br>1,2,3,<br>1,2,3,<br>1,2,3,<br>1,2,3,<br>1,2,3,<br>1,2,3,<br>1,2,3,<br>1,2,3,<br>1,2,3,<br>1,2,3,<br>1,2,3,<br>1,2,3,<br>1,2,3,<br>1,2,3,<br>1,2,3,<br>1,2,3,<br>1,2,3,<br>1,2,3,<br>1,2,3,<br>1,2,3,<br>1,2,3,<br>1,2,3,<br>1,2,3,<br>1,2,3,<br>1,2,3,<br>1,2,3,<br>1,2,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1,3,3,<br>1 | find the second the se | rol ae smal 7,8],k randon mallest                            | Dak  Dak  ,x);  m x0 fi   | oly<br>sitive in                                    | ateger s                                | such t   | hat k,  | d verify   | that is                                  | ndeed thates. Pic               | e –  | +01  | 115                                    | b  | OUI      | na   |      | oti      | vn_                  | 2           | Nu         | mk  | pers       | 5 :      | So   |             |                     |   |
| > s.  | SS(f, x) $SS(f, x)$  |  |  |  | $:=\frac{1}{361}$  | $-x^9 - \frac{3}{3}$  |   |   |  |   | [3, 5]   | ]  |                                 | 18124<br>361 x <sup>3</sup>  |  |  |  |          | V Ol | a fe | er<br>1m | tes-<br>inea         | +100<br>1 t | g a<br>Kat | ton | ono<br>esi | of<br>Wa | inte | ger<br>St ( | 5. I<br>1914        | c |
| > f  > ss  > or   | b(f, x, 5) $= Make$ $S(f, x)$ $b(f, x, 1)$ $rb(f, x, 3)$ $rb(f, x, 5)$ | .1, 1000,  | 1010)<br>[1.004<br>, 1010)<br>[2.999<br>, 1010)  | [5.0005] 3, 4, 5, 6, 7 f:= 146666, 0.  | 525, 4.99<br>(, 8), 2521<br>1 2521 x <sup>2</sup><br>9956004 | $ 94615, 5 $ $ 1, x) 9 - \frac{36}{252} $ $ 0, 1.0044 $ $ 2, 2.9999 $ | $\frac{1}{1}x^8 + \frac{1}{2}x^{15773} = 0.0005980$ | 546<br>2521 x <sup>7</sup><br>299560910 | 259, 5.0<br>- 453<br>252<br>0, 1.004<br>0, 2.999 | 0006803,<br>6/1 x <sup>6</sup> + 2/2<br>[44881, 0 | 4.999362<br>22449<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2521<br>2 | - 6728<br>2521<br>5, 1.0044<br>2, 2.9999 | $\frac{14}{1}x^4 + \frac{1}{2}$ | 996928, 3.0<br>93682, 5.00<br>18124<br>2521<br>x <sup>3</sup> –<br>562636, 1.1<br>000010, 3.1<br>999152, 4.1 | 006491,<br>- 1095<br>- 252<br>0004431<br>0000000 | 4.999288<br>684<br>11 x <sup>2</sup> + | 44, 5.0006<br>42841<br>2521 x<br>63496, 1. | 00442225 |      |      |          |                      |             |            |     |            |          |      |             | 5.I<br>Uqiu<br>25ZI |   |
| > Oi  | b(f, x, 7  | .1, 1000,  |  |  |  |   |   |   |  |   |  |  |                                 | 5529, 7.009  |  |  |  |          |      |      |          |                      |             |            |     |            |          |      |             |                     |   |
|   |  |  |  |  |  |   |   |   |  |   |  |  |                                 |  |  |  |  |          |      |      |          |                      |             |            |     |            |          |      |             |                     |   |
|   |  |  |  |  |  |   |   |   |  |   |  |  |                                 |  |  |  |  |          |      |      |          |                      |             |            |     |            |          |      |             |                     |   |
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|   |  |  |  |  |  |   |   |   |  |   |  |  |                                 |  |  |  |  |          |      |      |          |                      |             |            |     |            |          |      |             |                     |   |
|   |  |  |  |  |  |   |   |   |  |   |  |  |                                 |  |  |  |  |          |      |      |          |                      |             |            |     |            |          |      |             |                     |   |
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