Homework for Lecture 8 of Dr. Z.'s Dynamical Models in Biology class

Email the answers (as .pdf file) to

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by 8:00pm Monday, Oct. 6,, 2021.

Subject: hw8

with an attachment hw8FirstLast.pdf and/or hw8FirstLast.txt

Using

`http://sites.math.rutgers.edu/~zeilberg/Bio25/DMB8.txt`

1. Find ten random rational functions using `raRF(x,3)` .

For each of them find all the stable steady-states.

Then using `Orb(f,x,x0,2000,2020);` for `x0=-100.1, -10.1, 10.1, 100.1`

verify that the orbit always converges to that stable steady-state.

2. Find the terms between 2000-th and 2030-th terms of the **orbit** starting at $x_0 = 0.5$ of the recurrence

$$x_n = k x_{n-1}(1 - x_{n-1}) \quad ,$$

for $k = 1$, $k = 2$, $k = 2.5$, $k = 3.1$, $k = 3.2$, $k = 3.3$, $k = 3.5$. What do you find?

3. By Playing "High-Low" find the smallest positive integer such that `k`,

`f:=MakeNicePol([1,2,3,4,5,6,7,8],k,x);`

has two stable steady-states. Pick random `x0` from their neighborhood and verify that indeed the orbits converges to them.

Then keep upping k to find the smallest integer such that it has four stable steady-states. Pick random `x0` from their neighborhood and verify that indeed the orbits converges to them.

Daniyal Chaudhry HW 8

(1) a) $\frac{4x^3+9x^2+9x+5}{4x^3+6x^2+4x+3}$ Stable Steady State: Root Of $(4z^4+2z^3-5z^2-6z-5)$
All desired orbits go to [1.46, 1.46...]

b) $\frac{4x^3+5x^2+7x+5}{2x^3+10x^2+8x+6}$ SSS: Root Of $(2z^4+6z^3+3z^2-z-5)$
All orbits go to 0.7994

c) $\frac{8x^3+4x^2+8x+6}{10x^3+4x^2+9x+7}$ SSS: Root Of $(10z^4-4z^3+5z^2-z-6)$
All orbits go to 0.8696

d) $\frac{x^3+9x^2+2x+3}{4x^3+2x^2+5x+1}$ SSS: Root Of $(4z^4+z^3-4z^2-z-3)$
All orbits go to 1.197

e) $\frac{3x^3+6x^2+x+4}{x^3+2x^2+7x+1}$ SSS: Root Of $(z^4-z^3+z^2-4)$
All orbits go to 1.506

f) $\frac{10x^3+3x^2+10x+5}{8x^3+8x^2+6x+4}$ SSS: RootOf $(8z^4-2z^3+3z^2-6z-5)$
All orbits go to 1.0684

g) $\frac{x^3+6x^2+9x+7}{3x^3+5x^2+9x+2}$ SSS: RootOf $(3z^4+4z^3+3z^2-7z-7)$
orbits to 1.143

h) $\frac{9x^3+3x^2+5x+2}{3x^3+4x^2+5x+8}$ SSS: 2/3 (finally something nice)
orbits to 0.6667

i) $\frac{8x^3+3x^2+9x+3}{x^3+10x^2+9x+7}$ SSS: RootOf $(z^4+2z^3+6z^2-2z-3)$
orbits to 0.7471

j) $\frac{x^3+x^2+9x+5}{9x^3+9x^2+9x+3}$ SSS: Root of $(9z^4+8z^3+8z^2-6z-5)$
orbits to 0.7125

(2) Let $f(x)$ be $Kx(1-x)$ for $k = 1, 2, 2.5, 3.1, 3.2, 3.3, 3.5$

orbit at 0.5 for: x_0	entries 2000–2030
1	[0.000497, 0.000497..., 0.000490]
2	[0.5 ... 0.5]
2.5	[0.6 ... 0.6]
3.1	[0.558 ... 0.558]
3.2	[0.513 ... 0.513]
3.3	[0.479 ... 0.479]
3.5	[0.501, 0.874 ... 0.382]

all of these seem to converge to a clear steady state. for $k=1$, it seems to go to a very low value, while for $k=2$ until $k=3.3$, the SSS hovers around 0.5. for $k=3.5$, it seems to be weirdly unstable, oscillating between 0.382 & 0.874...

(3) $k=361$ is the first int. st there are 2 SSS: [3, 5]
orbit at 3.1 goes to 3.00 ✓ (Orb(f, x, 3.1, 2000, 2002) produces [3.000, 3.000, 3.000]. same method for other orbits
orbit at 5.1 goes to 5.00 ✓

$k=2521$ is the first int. st there are 4 SSS: [1, 3, 5, 7]
orbit at 1.1 goes to 1.00 ✓
at 3.1 $\longrightarrow$ 3 ✓
5.1 $\longrightarrow$ 5 ✓
7.1 $\longrightarrow$ 7 ✓