

Homework for Lecture 8 of Dr. Z.'s Dynamical Models in Biology class

Email the answers (as .pdf file) to

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by 8:00pm Monday, Oct. 6,, 2021.

Subject: hw8

with an attachment hw8FirstLast.pdf and/or hw8FirstLast.txt

Using

<http://sites.math.rutgers.edu/~zeilberg/Bio25/DMB8.txt>

1. Find ten random rational functions using `raRF(x,3)` .

For each of them find all the stable steady-states.

Then using `Orb(f,x,x0,2000,2020)`; for $x_0 = -100.1, -10.1, 10.1, 100.1$

verify that the orbit always converges to that stable steady-state.

2. Find the terms between 2000-th and 2030-th terms of the **orbit** starting at $x_0 = 0.5$ of the recurrence

$$x_n = kx_{n-1}(1 - x_{n-1}) \quad ,$$

for $k = 1, k = 2, k = 2.5, k = 3.1, k = 3.2, k = 3.3, k = 3.5$. What do you find?

3. By Playing “High-Low” find the smallest positive integer such that k ,

`f:=MakeNicePol([1,2,3,4,5,6,7,8],k,x);`

has two stable steady-states. Pick random x_0 from their neighborhood and verify that indeed the orbits converges to them.

Then keep upping k to find the smallest integer such that it has four stable steady-states. Pick random x_0 from their neighborhood and verify that indeed the orbits converges to them.

Problem 1

read "C:/Users/rlade/Downloads/DMB FILES/DMB8(1).txt" :

*For a list of the Main procedures type: Help8(); for help with a specific procedure type: Help6
(ProcedureName); for example Help6(RandomMC);*

*For a list of the other procedures type: Help8a(); for help with a specific procedure type: Help6
(ProcedureName); for example Help6(RandomMC);* **(1)**

for *i* **from** 1 **to** 10 **do**

f := raRF(*x*, 3);

print(`Function #`, *i*, ``, *f*);

stable_pts := evalf(SSS(*f*, *x*));

print("Stable steady-states:", *stable_pts*);

for *x0* **in** [-100.1, -10.1, 10.1, 100.1] **do**

orbit := Orb(*f*, *x*, *x0*, 2000, 2020);

print(*orbit*)

od;

print(`-----`);

od:

$$\text{Function \# 1, ``}, \frac{4x^3 + x^2 + 8x + 9}{3x^3 + 6x^2 + x + 8}$$

"Stable steady-states:", [1.176336839]

[1.176336838, 1.176336839, 1.176336838, 1.176336839, 1.176336838, 1.176336839,
1.176336838, 1.176336839, 1.176336838, 1.176336839, 1.176336838, 1.176336839,
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1.176336838, 1.176336839, 1.176336838]

[1.176336838, 1.176336839, 1.176336838, 1.176336839, 1.176336838, 1.176336839,
1.176336838, 1.176336839, 1.176336838, 1.176336839, 1.176336838, 1.176336839,
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1.176336838, 1.176336839, 1.176336838]

[1.176336839, 1.176336838, 1.176336839, 1.176336838, 1.176336839, 1.176336838,
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1.176336839, 1.176336838, 1.176336839]

[1.176336838, 1.176336839, 1.176336838, 1.176336839, 1.176336838, 1.176336839,

1.176336838, 1.176336839, 1.176336838, 1.176336839, 1.176336838, 1.176336839,
1.176336838, 1.176336839, 1.176336838, 1.176336839, 1.176336838, 1.176336839,
1.176336838, 1.176336839, 1.176336838]

$$\text{Function \# 2, } \therefore, \frac{2x^3 + 4x^2 + 9x + 10}{7x^3 + 2x^2 + 2x + 8}$$

"Stable steady-states:", [1.189633497]

[1.189633498, 1.189633496, 1.189633498, 1.189633496, 1.189633498, 1.189633496,
1.189633498, 1.189633496, 1.189633498, 1.189633496, 1.189633498, 1.189633496,
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[1.189633498, 1.189633496, 1.189633498, 1.189633496, 1.189633498, 1.189633496,
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1.189633498, 1.189633496, 1.189633498]

[1.189633498, 1.189633496, 1.189633498, 1.189633496, 1.189633498, 1.189633496,
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[1.189633498, 1.189633496, 1.189633498, 1.189633496, 1.189633498, 1.189633496,
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1.189633498, 1.189633496, 1.189633498]

$$\text{Function \# 3, } \therefore, \frac{7x^3 + 7x^2 + 6x + 9}{8x^3 + 7x^2 + 2x + 2}$$

"Stable steady-states:", [1.296250413]

[1.296250413, 1.296250413, 1.296250413, 1.296250413, 1.296250413, 1.296250413,
1.296250413, 1.296250413, 1.296250413, 1.296250413, 1.296250413, 1.296250413,
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1.296250413, 1.296250413, 1.296250413]

[1.296250413, 1.296250413, 1.296250413, 1.296250413, 1.296250413, 1.296250413,

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[1.296250413, 1.296250413, 1.296250413, 1.296250413, 1.296250413, 1.296250413,
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1.296250413, 1.296250413, 1.296250413]

[1.296250413, 1.296250413, 1.296250413, 1.296250413, 1.296250413, 1.296250413,
1.296250413, 1.296250413, 1.296250413, 1.296250413, 1.296250413, 1.296250413,
1.296250413, 1.296250413, 1.296250413, 1.296250413, 1.296250413, 1.296250413,
1.296250413, 1.296250413, 1.296250413]

$$\text{Function \# 4, `:`}, \frac{4x^3 + x^2 + x + 3}{9x^3 + 6x^2 + 9x + 10}$$

"Stable steady-states:", [0.2630144243]

[0.2630144244, 0.2630144242, 0.2630144244, 0.2630144242, 0.2630144244, 0.2630144242,
0.2630144244, 0.2630144242, 0.2630144244, 0.2630144242, 0.2630144244, 0.2630144242,
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[0.2630144244, 0.2630144242, 0.2630144244, 0.2630144242, 0.2630144244, 0.2630144242,
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[0.2630144244, 0.2630144242, 0.2630144244, 0.2630144242, 0.2630144244, 0.2630144242,
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[0.2630144244, 0.2630144242, 0.2630144244, 0.2630144242, 0.2630144244, 0.2630144242,
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0.2630144244, 0.2630144242, 0.2630144244]

$$\text{Function \# 5, '':} \frac{4x^3 + 8x^2 + 3x + 9}{9x^3 + 10x^2 + 10x + 3}$$

"Stable steady-states:", [0.8277954923]

[0.8277954930, 0.8277954917, 0.8277954930, 0.8277954917, 0.8277954930, 0.8277954917,
0.8277954930, 0.8277954917, 0.8277954930, 0.8277954917, 0.8277954930, 0.8277954917,
0.8277954930, 0.8277954917, 0.8277954930, 0.8277954917, 0.8277954930, 0.8277954917,
0.8277954930, 0.8277954917, 0.8277954930]

[0.8277954924, 0.8277954920, 0.8277954922, 0.8277954927, 0.8277954924, 0.8277954920,
0.8277954922, 0.8277954927, 0.8277954924, 0.8277954920, 0.8277954922, 0.8277954927,
0.8277954924, 0.8277954920, 0.8277954922, 0.8277954927, 0.8277954924, 0.8277954920,
0.8277954922, 0.8277954927, 0.8277954924]

[0.8277954930, 0.8277954917, 0.8277954930, 0.8277954917, 0.8277954930, 0.8277954917,
0.8277954930, 0.8277954917, 0.8277954930, 0.8277954917, 0.8277954930, 0.8277954917,
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[0.8277954930, 0.8277954917, 0.8277954930, 0.8277954917, 0.8277954930, 0.8277954917,
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0.8277954930, 0.8277954917, 0.8277954930]

$$\text{Function \# 6, '':} \frac{2x^3 + 4x^2 + 9x + 2}{9x^3 + 3x^2 + 4x + 8}$$

"Stable steady-states:", [0.7148038339]

[0.7148038339, 0.7148038343, 0.7148038339, 0.7148038343, 0.7148038339, 0.7148038343,
0.7148038339, 0.7148038343, 0.7148038339, 0.7148038343, 0.7148038339, 0.7148038343,
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[0.7148038339, 0.7148038343, 0.7148038339, 0.7148038343, 0.7148038339, 0.7148038343,
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[0.7148038343, 0.7148038339, 0.7148038343, 0.7148038339, 0.7148038343, 0.7148038339,

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0.7148038343, 0.7148038339, 0.7148038343]

[0.7148038343, 0.7148038339, 0.7148038343, 0.7148038339, 0.7148038343, 0.7148038339,
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0.7148038343, 0.7148038339, 0.7148038343]

$$\text{Function \#, 7, `:`}, \frac{x^3 + 5x^2 + 9x + 4}{4x^3 + 9x^2 + 9x + 2}$$

"Stable steady-states:", [0.8521796512]

[0.8521796512, 0.8521796512, 0.8521796512, 0.8521796512, 0.8521796512, 0.8521796512,
0.8521796512, 0.8521796512, 0.8521796512, 0.8521796512, 0.8521796512, 0.8521796512,
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[0.8521796512, 0.8521796512, 0.8521796512, 0.8521796512, 0.8521796512, 0.8521796512,
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[0.8521796512, 0.8521796512, 0.8521796512, 0.8521796512, 0.8521796512, 0.8521796512,
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0.8521796512, 0.8521796512, 0.8521796512]

[0.8521796512, 0.8521796512, 0.8521796512, 0.8521796512, 0.8521796512, 0.8521796512,
0.8521796512, 0.8521796512, 0.8521796512, 0.8521796512, 0.8521796512, 0.8521796512,
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0.8521796512, 0.8521796512, 0.8521796512]

$$\text{Function \#, 8, `:`}, \frac{4x^3 + 7x^2 + 6x + 2}{2x^3 + 9x^2 + 4x + 4}$$

"Stable steady-states:", [1.]

[1.0000000000, 1.0000000000, 1.0000000000, 1.0000000000, 1.0000000000, 1.0000000000,

[2.779538610, 2.779538610, 2.779538610, 2.779538610, 2.779538610, 2.779538610,
2.779538610, 2.779538610, 2.779538610, 2.779538610, 2.779538610, 2.779538610,

2.779538610, 2.779538610, 2.779538610, 2.779538610, 2.779538610, 2.779538610,
2.779538610, 2.779538610, 2.779538610]

$$\text{Function \#, 10, \text{'\`'}, \frac{x^3 + 6x^2 + 4x + 4}{6x^3 + 5x^2 + 8x + 8}}$$

"Stable steady-states:", [0.5485837703]

[0.5485837704, 0.5485837702, 0.5485837701, 0.5485837704, 0.5485837702, 0.5485837701,
0.5485837704, 0.5485837702, 0.5485837701, 0.5485837704, 0.5485837702, 0.5485837701,
0.5485837704, 0.5485837702, 0.5485837701, 0.5485837704, 0.5485837702, 0.5485837701,
0.5485837704, 0.5485837702, 0.5485837701]

[0.5485837704, 0.5485837702, 0.5485837701, 0.5485837704, 0.5485837702, 0.5485837701,
0.5485837704, 0.5485837702, 0.5485837701, 0.5485837704, 0.5485837702, 0.5485837701,
0.5485837704, 0.5485837702, 0.5485837701, 0.5485837704, 0.5485837702, 0.5485837701,
0.5485837704, 0.5485837702, 0.5485837701]

[0.5485837702, 0.5485837701, 0.5485837704, 0.5485837702, 0.5485837701, 0.5485837704,
0.5485837702, 0.5485837701, 0.5485837704, 0.5485837702, 0.5485837701, 0.5485837704,
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0.5485837702, 0.5485837701, 0.5485837704]

[0.5485837704, 0.5485837702, 0.5485837701, 0.5485837704, 0.5485837702, 0.5485837701,
0.5485837704, 0.5485837702, 0.5485837701, 0.5485837704, 0.5485837702, 0.5485837701,
0.5485837704, 0.5485837702, 0.5485837701, 0.5485837704, 0.5485837702, 0.5485837701,
0.5485837704, 0.5485837702, 0.5485837701]

Problem 2

read "C:/Users/rlade/Downloads/DMB FILES/DMB8(1).txt" :

For a list of the Main procedures type: Help8(); for help with a specific procedure type: Help6 (ProcedureName); for example Help6(RandomMC);

For a list of the other procedures type: Help8a()); for help with a specific procedure type: Help6 (ProcedureName); for example Help6(RandomMC); (1)

Help8(Orb)

Orb(f,x,x0,K1,K2): Given a function f of the variable x, an initial value x(0) finds the portion of the orbit

of the recurrence $a(n+1)=f(a(n))$, in other words $[a(K1),a(K1+1), \dots, a(K2)]$. If you want the full orbit take $K1=1$.

For example, try:

*Orb(5/2*x*(1-x),x,0.2,1000,1010);* (2)

$$f := x \cdot (1 - x);$$

$$f := x (1 - x) \quad (3)$$

Orb(f, x, 0.5, 2000, 2030);

[0.0004976675995, 0.0004974199265, 0.0004971724999, 0.0004969253194, 0.0004966783846, (4)

0.0004964316952, 0.0004961852508, 0.0004959390510, 0.0004956930954, 0.0004954473838,

0.0004952019157, 0.0004949566908, 0.0004947117087, 0.0004944669690, 0.0004942224714,

0.0004939782155, 0.0004937342010, 0.0004934904275, 0.0004932468947, 0.0004930036022,

0.0004927605496, 0.0004925177367, 0.0004922751630, 0.0004920328281, 0.0004917907318,

0.0004915488737, 0.0004913072534, 0.0004910658706, 0.0004908247249, 0.0004905838160,

0.0004903431435] $\hookrightarrow$ diverges away from 0.5, so pop. goes extinct. (unstable ss)

$$f := 2 \cdot x \cdot (1 - x);$$

$$f := 2 x (1 - x) \quad (5)$$

Orb(f, x, 0.5, 2000, 2030);

[0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, (6)

0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000,

0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000,

0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000,

0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000,

0.5000000000] $\rightarrow$ Converges to 0.5, population stabilizes (stable ss)

$$f := 2.5 \cdot x \cdot (1 - x);$$

$$f := 2.5 x (1 - x) \quad (7)$$

$Orb(f, x, 0.5, 2000, 2030);$

[0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, (8)

0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000,
0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000,
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0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000,
0.6000000000]

↳ Converges to 0.6, population stabilizes
(stable SS) → at a higher value but still stable

$f := 3.1 \cdot x \cdot (1 - x);$

$f := 3.1 x (1 - x)$ (9)

$Orb(f, x, 0.5, 2000, 2030);$

[0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, (10)

0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203,
0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203,
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0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203,
0.5580141245]

↳ a period 2 oscillation (system no longer
settles at a single pt) btwn (0.558 and 0.765)

$f := 3.2 \cdot x \cdot (1 - x);$

$f := 3.2 x (1 - x)$ (11)

$Orb(f, x, 0.5, 2000, 2030);$

[0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, (12)

0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906,
0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906,
0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906,
0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906,
0.5130445091]

↳ still a period 2 oscillation behavior
btwn (0.513 and 0.799)

$f := 3.3 \cdot x \cdot (1 - x);$

$f := 3.3 x (1 - x)$ (13)

$Orb(f, x, 0.5, 2000, 2030);$

[0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, (14)

0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832,
0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832,
0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832,
0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832,

0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832,
0.4794270198]

↳ period 2 oscillation behavior continues, just larger amplitude

$f := 3.5 \cdot x \cdot (1 - x);$

$f := 3.5 x (1 - x)$

(15)

$Orb(f, x, 0.5, 2000, 2030);$

[0.5008842111, 0.8749972637, 0.3828196827, 0.8269407062, 0.5008842111, 0.8749972637,
0.3828196827, 0.8269407062, 0.5008842111, 0.8749972637, 0.3828196827, 0.8269407062,
0.5008842111, 0.8749972637, 0.3828196827, 0.8269407062, 0.5008842111, 0.8749972637,
0.3828196827, 0.8269407062, 0.5008842111, 0.8749972637, 0.3828196827, 0.8269407062,
0.5008842111, 0.8749972637, 0.3828196827, 0.8269407062, 0.5008842111, 0.8749972637,
0.3828196827]

(16)

↳ period 4 oscillation behavior,
first sign of chaotic transition.

as K increases, the behavior of the funct. is: stability $\rightarrow$ oscillatory $\rightarrow$ complex/chaotic dynamics.

Problem 3

read "C:/Users/rlade/Downloads/DMB FILES/DMB8(1).txt" :

*For a list of the Main procedures type: Help8(); for help with a specific procedure type: Help6
(ProcedureName); for example Help6(RandomMC);*

*For a list of the other procedures type: Help8a(); for help with a specific procedure type: Help6
(ProcedureName); for example Help6(RandomMC);* **(1)**

$f := \text{MakeNicePol}([1, 2, 3, 4, 5, 6, 7, 8], 1000, x);$
 $\text{SSS}(f, x)$

$$f := \frac{1}{1000} x^9 - \frac{9}{250} x^8 + \frac{273}{500} x^7 - \frac{567}{125} x^6 + \frac{22449}{1000} x^5 - \frac{16821}{250} x^4 + \frac{29531}{250} x^3$$

$$- \frac{13698}{125} x^2 + \frac{1033}{25} x$$

$[3, 5]$ **(2)**

$f := \text{MakeNicePol}([1, 2, 3, 4, 5, 6, 7, 8], 250, x);$
 $\text{SSS}(f, x)$

$$f := \frac{1}{250} x^9 - \frac{18}{125} x^8 + \frac{273}{125} x^7 - \frac{2268}{125} x^6 + \frac{22449}{250} x^5 - \frac{33642}{125} x^4 + \frac{59062}{125} x^3$$

$$- \frac{54792}{125} x^2 + \frac{4057}{25} x$$

$[]$ **(3)**

$f := \text{MakeNicePol}([1, 2, 3, 4, 5, 6, 7, 8], 300, x);$
 $\text{SSS}(f, x)$

$$f := \frac{1}{300} x^9 - \frac{3}{25} x^8 + \frac{91}{50} x^7 - \frac{378}{25} x^6 + \frac{7483}{100} x^5 - \frac{5607}{25} x^4 + \frac{29531}{75} x^3 - \frac{9132}{25} x^2$$

$$+ \frac{677}{5} x$$

$[]$ **(4)**

$f := \text{MakeNicePol}([1, 2, 3, 4, 5, 6, 7, 8], 350, x);$
 $\text{SSS}(f, x)$

$$f := \frac{1}{350} x^9 - \frac{18}{175} x^8 + \frac{39}{25} x^7 - \frac{324}{25} x^6 + \frac{3207}{50} x^5 - \frac{4806}{25} x^4 + \frac{59062}{175} x^3 - \frac{54792}{175} x^2$$

$$+ \frac{581}{5} x$$

$[]$ **(5)**

$f := \text{MakeNicePol}([1, 2, 3, 4, 5, 6, 7, 8], 400, x);$

$SSS(f, x)$

$$f := \frac{1}{400} x^9 - \frac{9}{100} x^8 + \frac{273}{200} x^7 - \frac{567}{50} x^6 + \frac{22449}{400} x^5 - \frac{16821}{100} x^4 + \frac{29531}{100} x^3 - \frac{6849}{25} x^2 + \frac{509}{5} x$$

$[3, 5]$

(6)

for k **from** 300 **to** 400 **by** 1 **do**

$f := \text{MakeNicePol}([1, 2, 3, 4, 5, 6, 7, 8], k, x);$

$\text{print}(k, \text{"Stable SS="}, SSS(f, x));$

od:

300, "Stable SS =", []

301, "Stable SS =", []

302, "Stable SS =", []

303, "Stable SS =", []

304, "Stable SS =", []

305, "Stable SS =", []

306, "Stable SS =", []

307, "Stable SS =", []

308, "Stable SS =", []

309, "Stable SS =", []

310, "Stable SS =", []

311, "Stable SS =", []

312, "Stable SS =", []

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315, "Stable SS =", []

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347, "Stable SS =", []

348, "Stable SS =", []
349, "Stable SS =", []
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354, "Stable SS =", []
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356, "Stable SS =", []
357, "Stable SS =", []
358, "Stable SS =", []
359, "Stable SS =", []
360, "Stable SS =", []
361, "Stable SS =", [3, 5]
362, "Stable SS =", [3, 5]
363, "Stable SS =", [3, 5]
364, "Stable SS =", [3, 5]
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366, "Stable SS =", [3, 5]
367, "Stable SS =", [3, 5]
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 383, "Stable SS =", [3, 5]
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 394, "Stable SS =", [3, 5]
 395, "Stable SS =", [3, 5]
 396, "Stable SS =", [3, 5]
 397, "Stable SS =", [3, 5]
 398, "Stable SS =", [3, 5]
 399, "Stable SS =", [3, 5]
 400, "Stable SS =", [3, 5]

(7)

#At around K=361 we see the first two Stable SS

#We test to make sure they are stable.

Orb(f, x, 3.1, 1000, 1020);

[3.0000043, 2.9999957, 3.0000043, 2.9999957, 3.0000043, 2.9999957, 3.0000043, 2.9999957,
 3.0000043, 2.9999957, 3.0000043, 2.9999957, 3.0000043, 2.9999957, 3.0000043, 2.9999957,
 3.0000043, 2.9999957, 3.0000043, 2.9999957, 3.0000043]

(8)

Orb(f, x, 5.1, 1000, 1020);

[5.0001384, 4.9999331, 4.9999786, 5.0000785, 5.0000003, 5.0001485, 4.9999813, 5.0000573,

(9)

4.9999441, 5.0001634, 4.9998151, 5.0001612, 4.9997982, 5.0001568, 4.9998472, 5.0001200,
4.9997110, 5.0002418, 4.9997732, 5.0002498, 4.9996606]

#test points of K to see where there becomes 4 Stable SS

$f := \text{MakeNicePol}([1, 2, 3, 4, 5, 6, 7, 8], 2800, x);$

$\text{SSS}(f, x);$

$$f := \frac{1}{2800} x^9 - \frac{9}{700} x^8 + \frac{39}{200} x^7 - \frac{81}{50} x^6 + \frac{3207}{400} x^5 - \frac{2403}{100} x^4 + \frac{29531}{700} x^3 - \frac{6849}{175} x^2 + \frac{77}{5} x$$

[1, 3, 5, 7]

(10)

$f := \text{MakeNicePol}([1, 2, 3, 4, 5, 6, 7, 8], 2500, x);$

$\text{SSS}(f, x);$

$$f := \frac{1}{2500} x^9 - \frac{9}{625} x^8 + \frac{273}{1250} x^7 - \frac{1134}{625} x^6 + \frac{22449}{2500} x^5 - \frac{16821}{625} x^4 + \frac{29531}{625} x^3 - \frac{27396}{625} x^2 + \frac{2141}{125} x$$

[3, 5]

(11)

for k from 2500 to 2700 by 1 do

$f := \text{MakeNicePol}([1, 2, 3, 4, 5, 6, 7, 8], k, x);$

$\text{print}(k, \text{"Stable SS ="}, \text{SSS}(f, x));$

od:

2500, "Stable SS =", [3, 5]

2501, "Stable SS =", [3, 5]

2502, "Stable SS =", [3, 5]

2503, "Stable SS =", [3, 5]

2504, "Stable SS =", [3, 5]

2505, "Stable SS =", [3, 5]

2506, "Stable SS =", [3, 5]

2507, "Stable SS =", [3, 5]

2508, "Stable SS =", [3, 5]

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2511, "Stable SS =", [3, 5]

2512, "Stable SS =", [3, 5]
2513, "Stable SS =", [3, 5]
2514, "Stable SS =", [3, 5]
2515, "Stable SS =", [3, 5]
2516, "Stable SS =", [3, 5]
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2521, "Stable SS =", [1, 3, 5, 7]
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2524, "Stable SS =", [1, 3, 5, 7]
2525, "Stable SS =", [1, 3, 5, 7]
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2692, "Stable SS =", [1, 3, 5, 7]
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2697, "Stable SS =", [1, 3, 5, 7]

2698, "Stable SS =", [1, 3, 5, 7]

2699, "Stable SS =", [1, 3, 5, 7]

2700, "Stable SS =", [1, 3, 5, 7]

(12)

#At around K=2521 we see the four Stable SS

#We test make sure they are stable

Orb(f, x, 1.1, 1000, 1020);

Orb(f, x, 3.1, 1000, 1020);

Orb(f, x, 5.1, 1000, 1020) Orb(f, x, 7.1, 1000, 1020);

[7.00003218, 6.99996922, 7.00002029, 6.99993351, 6.99993080, 7.00003180, 7.00000771,

(13)

7.00003644, 6.99999752, 6.99998641, 6.99995090, 6.99999926, 6.99998347, 6.99995778,

7.00003274, 7.00006238, 7.00002287, 6.99996132, 7.00000040, 7.00005699, 7.00004414]