

hw 7

hw 7 zachariah A

i) $x = rx(1-x)$
 $x - rx(1-x) = 0$
 $x(1-r(1-x)) = 0$
 $x = 0$ $1-r+r x = 0$

$rx = r-1$
 $x = 1 - \frac{1}{r}$

So steady states 0 and 0.6

ii) $f(x) = rx(1-x) = rx - x^2$
 $f'(x) = r - 2x$
 $f'(0) = r(1-2x)$

$r = 2.5$ $x = 0$ $f'(x) = 2.5 > 0$ so unstable at 0
 $r = 2.5$ $x = 0.6$ $|f'(x)| = |2.5(1-1.2)| = 0.5 < 1$
 So stable at 0.6

iii) Orbit n

0	1	2	3	4	5	6	7	8	9
0.001	0.0024	0.0061	0.015	0.033	0.061	0.104	0.160	0.221	0.281
0.61	0.59	0.60	0.59	0.60	0.59	0.60	0.59	0.60	0.6

So even when close to 0, the orbit converged to 0.6.

i) $\{0, \frac{19}{29}\}$

ii) $r = 2.9$ $x = 0$ $|f'(x)| = 2.9 > 1$ unstable at 0
 $r = 2.9$ $x = \frac{19}{29}$ $|f'(x)| = |2.9(1 - \frac{19}{29})| = 0.9 < 1$
 2.9 is stable

iii) Orbit n

0	1	2	3	4	5	6	7	8	9
-0.1	-0.319	-1.22	-7.86	-701	-118660	-4x10 ¹⁰	-5x10 ²¹	-6x10 ³⁷	-1x10 ⁵³
0.61	0.689	0.62	0.682	0.627	0.677	0.633	0.673	0.632	0.66

i) $\{0, \frac{21}{31}\}$ Diverges from near 0, converges towards $\frac{19}{29} \approx 0.655$ from 0.61
 ii) $|f'(x)| = \frac{31}{10} (1 - \frac{21x^2}{31}) = \frac{11}{10} > 1$
 unstable at $\frac{21}{31}$
 so unstable at 0.

Orbit

0	1	2	3	4	5	6	7	8	9
0.021	0.003	0.009	0.029	0.088	0.249	0.581	0.754	0.573	0.758
0.6	0.744	0.578	0.750	0.578	0.754	0.574	0.756	0.572	0.757

Neither seem to converge.

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2. a. i) $\frac{1}{5}(1)^3 - \frac{5}{4}(1)^2 + \frac{39}{8}(1) - \frac{15}{4} = 0$ ✓
 $\frac{1}{5}(2)^3 - \frac{5}{4}(2)^2 + \frac{39}{8}(2) - \frac{15}{4} = 0$ ✓
 $\frac{1}{5}(3)^3 - \frac{5}{4}(3)^2 + \frac{39}{8}(3) - \frac{15}{4} = 0$ ✓
 ii) $f(x) = \frac{1}{5}x^3 - \frac{5}{4}x^2 + \frac{39}{8}x - \frac{15}{4}$

2. a. i) $f(x) = \frac{1}{5}x^3 - \frac{6}{5}x^2 + \frac{16}{5}x - \frac{6}{5}$
 $f(1) = 1$ ✓ $f(2) = 2$ ✓ $f(3) = 3$ ✓

ii) $f'(x) = \frac{3}{5}x^2 - \frac{12}{5}x + \frac{16}{5}$
 $f'(1) = 1.4 > 1$ so unstable at 1
 $f'(2) = 0.8 < 1$ stable at 2
 $f'(3) = 1.4 > 1$ unstable at 3.

iii) orbit

x	1	2	3	4	5	6	7	8	9
1.1	1.13	1.17	1.23	1.37	1.46	1.56	1.67	1.78	1.88
2.1	2.07	2.05	2.04	2.03	2.03	2.027	2.024	2.022	2.021
3.1	3.14	3.23	3.37	3.57	3.84	4.19	4.45	4.71	4.87

Evidently all but 2 diverge. Clearly there is attraction close to 2 but not 1 and 3.

b. i) $f(x) = \frac{1}{8}x^3 - \frac{5}{4}x^2 + \frac{39}{8}x - \frac{15}{4}$
 $f(2) = 2$ ✓ $f(3) = 3$ ✓ $f(5) = 5$ ✓

ii) $f'(x) = \frac{3}{8}x^2 - \frac{5}{2}x + \frac{39}{8}$
 $f'(2) = 1.375 > 1$ unstable at 2
 $f'(3) = 0.75 < 1$ stable at 3
 $f'(5) = 1.75 > 1$ unstable at 5

(ii)

Orbit

	1	2	3	4	5	6	7	8	9
0									
2.1	2.11	2.13	2.15	2.14	2.23	2.27	2.32	2.37	2.43
3.1	3.08	3.06	3.04	3.03	3.02	3.01	3.01	3.008	3.005
5.1	4.88	4.6	3.77	3.78	3.044	3.07	3.000073	3.0	3.0

Clearly runs from 2 and 5 and is attracted towards 3.